**Section 5.2 - The Definite Integral** - p. 324 Stewart, 4th Ed.
Instructor edition Recall: area under a curve =

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x,$$

where $x_i^*$ is any $x$-value you want between $x_{i-1}$ and $x_i$. The sum $\sum_{i=1}^{n} f(x_i^*)\Delta x$ is called a **Riemann sum**.

The limit of the Riemann sum (the area) is called the **definite integral** of $f(x)$ from $a$ to $b$, and it is denoted $\int_a^b f(x)\,dx$. We read: "The integral from $a$ to $b$ of $f(x)$, $dx$." $a$ is called the **lower limit** of integration; $b$ is called the **upper limit**. $f(x)$ is called the **integrand**, and $dx$ is sometimes called the **differential**.

Example 1: I asserted that $\int_1^4 x^2 dx = 21$.

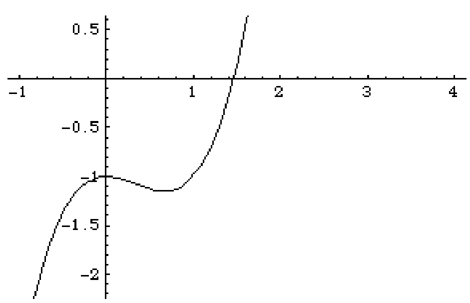
Example 2: $\int_{-2}^{3} 10\,dx =$ ______________________________

Note that $\int_a^b f(x)dx$ is a *number* — it's an *area*, after all. So it does not depend on $x$:

$$\int_a^b f(x)\,dx = \int_a^b f(t)dt = \int_a^b f(@)d@, \text{ etc.}$$

So far all of our integrands have been *positive* from $a$ to $b$ (above the $x$-axis). How do we interpret the area "under" a curve if the curve is *below* the $x$-axis?
Answer: it counts as *negative area.*

Example: Note that $f(x) = x^3 - x^2 - 1$ is *negative* on the interval $[-1, 1]$:



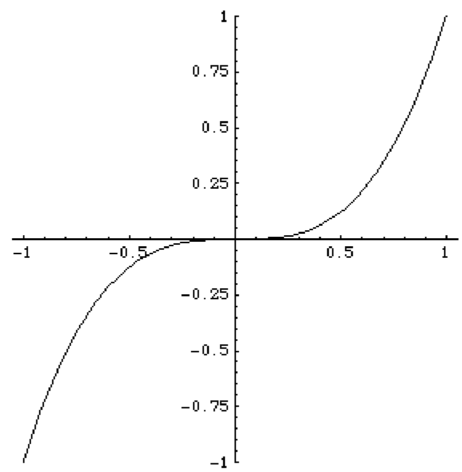

It turns out that

$$\int_{-1}^{1} x^3 - x^2 - 1\,dx = -\frac{8}{3}$$

(we will learn how to compute this).

Example: $\int_{-2}^{2} x^3\,dx = 0$, since there is as much negative area as positive.

We *add* the areas above the $x$-axis, and *subtract* the areas below.

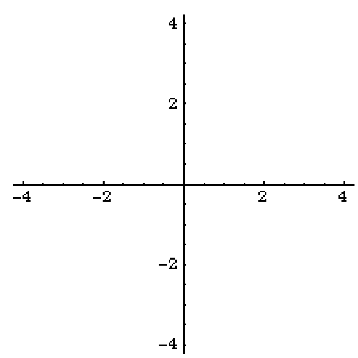



Example: Evaluate

$$\int_{-2}^{1} (x+1)\, dx$$

by interpreting in terms of areas.

Solution: Draw a picture:




We see that positive area = ______, negative area = ______. Therefore the integral is equal to

________________________________________.