Math 76 Practice Problems for Midterm II - Solutions
§§7.2-10.3

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

Multiple Choice. *Circle the letter of the best answer.*

1. $\displaystyle\int_{-\pi/4}^{\pi/4} \tan^2 x\, dx =$

(a) $1+\dfrac{\pi}{2}$

(b) $1-\dfrac{\pi}{4}$

(c) $\boxed{2-\dfrac{\pi}{2}}$

(d) $2+\dfrac{\pi}{4}$

Using the Pythagorean identity $\tan^2 x = \sec^2 x - 1$, we have

$$\begin{aligned}\int_{-\pi/4}^{\pi/4} \tan^2 x\, dx &= \int_{-\pi/4}^{\pi/4} (\sec^2 x - 1)\, dx \\ &= \tan x - x \Big|_{-\pi/4}^{\pi/4} \\ &= \left(\tan\left(\tfrac{\pi}{4}\right) - \tfrac{\pi}{4}\right) - \left(\tan\left(-\tfrac{\pi}{4}\right) + \tfrac{\pi}{4}\right) \\ &= 1 - \tfrac{\pi}{4} - (-1) - \tfrac{\pi}{4} = \boxed{2 - \tfrac{\pi}{2}}\end{aligned}$$

2. The partial fraction decomposition of $\dfrac{4x^3-2x+1}{(x^2+5)(x-3)^2}$ is

(a) $\dfrac{A}{x^2+5}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$

(b) $\boxed{\dfrac{Ax+B}{x^2+5}+\dfrac{C}{x-3}+\dfrac{D}{(x-3)^2}}$

(c) $\dfrac{Ax+B}{x^2+5}+\dfrac{C}{(x-3)^2}$

(d) $\dfrac{Ax+B}{x^2+5}+\dfrac{C}{x-3}+\dfrac{Dx+E}{(x-3)^2}$

In the denominator we have one irreducible quadratic factor x^2+5, so we put a linear form in the numerator of that term. We also have a repeated linear factor $(x-3)^2$, so we put a constant form in the numerator of each power of $x-3$ up to the maximum $(x-3)^2$.

3. $\displaystyle\int \frac{2x-1}{(x+1)(x-2)}\, dx =$

(a) $\boxed{\ln|x+1| + \ln|x-2| + C}$

(b) $3\ln|x+1| - 2\ln|x-2| + C$

(c) $\ln|x+1| - \ln|x-2| + C$

(d) $-\ln|x+1| + \ln|x-2| + C$

If $\dfrac{2x-1}{(x+1)(x-2)} = \dfrac{A}{x+1}+\dfrac{B}{x-2}$, then $A(x-2)+B(x+1) = 2x-1$. Setting $x=2$ we see that

$3B = 3$, so $B = 1$. Setting $x = -1$, we get $-3A = -3$, so $A = 1$. Therefore

$$\begin{aligned}\int \frac{2x-1}{(x+1)(x-2)}\,dx &= \int \frac{1}{x+1} + \frac{1}{x-2}\,dx \\ &= \ln|x+1| + \ln|x-2| + C.\end{aligned}$$

4. $\displaystyle\int_0^e \ln x\,dx =$

(a) 1

(b) $\boxed{0}$

(c) ∞

(d) $-\infty$

This is an improper integral. We have

$$\begin{aligned}\int_0^e \ln x\,dx &= \lim_{t\to 0^+} \int_t^e \ln x\,dx \\ &= \lim_{t\to 0^+} x\ln x - x \Big|_t^e \qquad \text{(using integration by parts)} \\ &= \lim_{t\to 0^+} (e-e) - (t\ln t - t) \\ &= \lim_{t\to 0^+} -t\ln t + t \\ &= \lim_{t\to 0^+} -t\ln t\end{aligned}$$

Recall from Math 75 that this is an indeterminate form of type $0 \cdot -\infty$. So we use l'Hôpital's Rule as follows:

$$\begin{aligned}&= \lim_{t\to 0^+} -\frac{\ln t}{\frac{1}{t}} \\ &= \lim_{t\to 0^+} \frac{\frac{1}{t}}{\frac{1}{t^2}} = \lim_{t\to 0^+} t = \boxed{0}\end{aligned}$$

5. The length of the curve $x = y^3 - y$ from $y = 1$ to $y = 3$ is

(a) $\displaystyle 2\pi \int_1^3 \sqrt{1+(3y^2-1)^2}\,dy$

(b) $\displaystyle \boxed{\int_1^3 \sqrt{9y^4-6y^2+2}\,dy}$

(c) $\displaystyle \int_1^3 \sqrt{1+y^3+y}\,dy$

(d) $\displaystyle \int_1^3 \sqrt{3y^2}\,dy$

We have $x' = 3y^2 - 1$, so $(x')^2 = 9y^4 - 6y^2 + 1$. Therefore the arc length is as given above.

6. The area of the surface formed by rotating the curve $x = y^3 - y$ from $y = 1$ to $y = 3$ about the x-axis is

(a) $\displaystyle \boxed{2\pi \int_1^3 y\sqrt{1+(3y^2-1)^2}\,dy}$

(b) $\displaystyle \int_1^3 \sqrt{9y^4-6y^2+2}\,dy$

(c) $\displaystyle 2\pi \int_1^3 (y^3-y)\sqrt{1+y^3-y}\,dy$

(d) $\displaystyle 2\pi \int_1^3 y\sqrt{3y^2}\,dy$

As in #5 above we have $(x')^2 = (3y^2-1)^2$. We are rotating about the x-axis and thus the "radius" element in our formula will be y. Therefore the surface area is $2\pi \int_1^3 y\sqrt{1+(3y^2-1)^2}\,dy$.

7. A trough is filled with water. The ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. The hydrostatic force on one end of the trough is

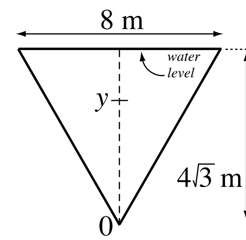
(a) $\frac{9800\sqrt{3}}{2}\int_0^{4\sqrt{3}} y(y-8)\,dy$

(b) $\frac{9800}{\sqrt{3}}\int_0^{4\sqrt{3}} y^2\,dy$

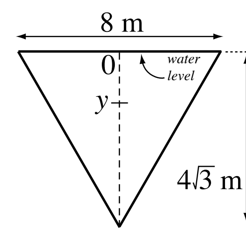
(c) $9800\int_0^{8}(8-y)y\,dy$

(d) $\boxed{\frac{19600}{\sqrt{3}}\int_0^{4\sqrt{3}}(4\sqrt{3}-y)y\,dy}$

The triangle is shown. Using the Pythagorean Theorem the height of the triangle is $4\sqrt{3}$ m, so putting the origin at the bottom vertex of the triangle we have that the depth at y is $d(y) = 4\sqrt{3} - y$. Using similar triangles we also find that $w(y) = \frac{2y}{\sqrt{(3)}}$. So we have $F = 9800\int_0^{4\sqrt{3}}(4\sqrt{3}-y)\frac{2y}{\sqrt{(3)}}\,dy = \frac{19600}{\sqrt{3}}\int_0^{4\sqrt{3}}(4\sqrt{3}-y)y\,dy$.



One note: if you put the origin somewhere else your integral will look different, but you should still be able to wiggle it (through algebra, substitution, or other means) into something that matches the above. For instance, if we put the origin at the top of the triangle (as at right), then $d(y) = y$ and $w(y) = \frac{2}{\sqrt{3}}(4\sqrt{3}-y)$, which simplifies to $w(y) = 8 - \frac{2}{\sqrt{3}}y$. The integral still goes from 0 to $4\sqrt{3}$, as it turns out.



Therefore we get $F = 9800\int_0^{4\sqrt{3}}\left(8-\frac{2}{\sqrt{3}}y\right)y\,dy = 9800\int_0^{4\sqrt{3}}\frac{2}{\sqrt{3}}\left(4\sqrt{3}-y\right)y\,dy$, which is now the same as the answer in the multiple choice.

8. The coordinates of the center of mass of the region enclosed by $y = x$, $y = x-4$, $x = 1$ and $x = 3$ are

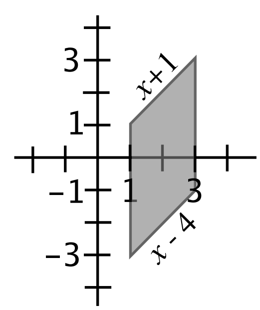
(a) $(2.5, 0.5)$

(b) $(1.75, 0)$

(c) $\boxed{(2, 0)}$

(d) $(2, 0.5)$

A picture of the region is shown. It is a parallelogram. By inspection, we can see by a kind of "skew symmetry" that the coordinates must be as above. But we can also compute them using the formulas: the area of a parallelogram is the base times the height, so $4\cdot 2 = 8$ (or you can do the integral $\int_1^3 (x-(x-4))\,dx$). So the coordinates are



$$\begin{aligned}\overline{x} &= \frac{1}{8}\int_1^3 x(x-(x-4))\,dx \\ &= \frac{1}{8}\int_1^3 x(4)\,dx = \frac{1}{8}\int_1^3 4x\,dx \\ &= \frac{1}{8}2x^2\,\Big|_1^3 = \frac{1}{8}(18-2) = \frac{16}{8} = 2\end{aligned}$$

and

$$\begin{aligned}\overline{y} &= \frac{1}{8}\cdot\frac{1}{2}\int_1^3 (x^2-(x-4)^2)\,dx \qquad \text{(use caution here; take careful note of the formula!)} \\ &= \frac{1}{16}\int_1^3 x^2-(x^2-8x+16)\,dx \\ &= \frac{1}{16}\int_1^3 8x-16\,dx \\ &= \frac{1}{16}(4x^2-16x)\,\Big|_1^3 \\ &= \frac{1}{16}((36-48)-(4-16)) = 0.\end{aligned}$$

9. The equation of the line tangent to the curve $\begin{matrix} x = e^{\sqrt{t}} \\ y = t - \ln t^2 \end{matrix}$ at the point corresponding to $t = 4$ is

(a) $y = \frac{2}{e^2}x + 4 - \ln 16$

(b) $y = \frac{1}{2}x + 4 - \ln 16 - \frac{1}{2}e^2$

(c) $y = \frac{e^2}{4}x + 4 - \ln 16 - \frac{1}{4}e^4$

(d) $\boxed{y = \frac{2}{e^2}x + 2 - \ln 16}$

The slope is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ evaluated at $t = 4$. Since $\frac{dy}{dt} = 1 - \frac{2}{t} = \frac{t-2}{t}$ and $\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}e^{\sqrt{t}} = \frac{e^{\sqrt{t}}}{2\sqrt{t}}$, we have $\frac{dy}{dx} = \frac{t-2}{t}\cdot\frac{2\sqrt{t}}{e^{\sqrt{t}}} = \frac{2(t-2)\sqrt{t}}{te^{\sqrt{t}}}$, which evaluated at $t = 4$ is $\frac{2}{e^2}$. Now the point on the curve corresponding to $t = 4$ (the point of tangency) is $(e^2, 4-\ln 16)$, so $4-\ln 16 = \frac{2}{e^2}e^2 + b = 2 + b$. Therefore $b = 2 - \ln 16$, and the equation of the line is $y = \frac{2}{e^2}x + 2 - \ln 16$.

10. In polar coordinates, the point $\left(3, \frac{\pi}{2}\right)$ represents the same location as the point

(a) $\boxed{\left(-3, -\frac{\pi}{2}\right)}$

(b) $\left(3, -\frac{\pi}{2}\right)$

(c) $\left(-3, \frac{\pi}{2}\right)$

(d) $\left(3, \frac{3\pi}{2}\right)$

The only two ways to get different names for the same polar point are

- Replace θ by a co-terminal angle (another angle in the same direction as θ, e.g. $\frac{\pi}{2}$ and $-\frac{3\pi}{2}$);
- Change r to $-r$ and change the angle θ to one pointing in the opposite direction.

Since neither $-\frac{\pi}{2}$ nor $\frac{3\pi}{2}$ is co-terminal to $\frac{\pi}{2}$, (b) and (d) are not correct. (c) is incorrect since r has been changed to $-r$ but the angle has not changed. Only (a) has r changed to $-r$ and the angle changed to one pointing in the opposite direction.

Fill-In.

1. $\displaystyle\int \sec^3 x\, dx = \underline{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C}$.

   We did this problem in class. To recap: use parts and solve for the integral. Please see me if you would like to go over this important technique.

2. $\displaystyle\int \sin^3 x\, dx = \underline{-\cos x + \tfrac{1}{3}\cos^3 x + C}$.

   We have

$$\begin{aligned}\int \sin^3 x\, dx &= \int \sin^2 x \sin x\, dx \\ &= \int (1-\cos^2 x)\sin x\, dx \qquad \text{(Let } u = \cos x\text{. Then } du = -\sin x\, dx) \\ &= -\int (1-u^2)\, du \\ &= -\left(u - \tfrac{1}{3}u^3\right) + C \\ &= -\cos x + \tfrac{1}{3}\cos^3 x + C.\end{aligned}$$

3. To evaluate the integral $\displaystyle\int \sqrt{5+x^2}\, dx$, it is best to use the trigonometric substitution

   $x = \underset{\text{(function of } \theta)}{\underline{\sqrt{5}\tan\theta}}$.

   For the form $\sqrt{a^2+x^2}$ it is best to use the trigonometric substitution $x = a\tan\theta$, where it is understood that $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. To see why, notice that the integral above becomes

$$\int \sqrt{5+5\tan^2\theta}\sqrt{5}\sec^2\theta\, d\theta = 5\int \sqrt{1+\tan^2\theta}\sec^2\theta\, d\theta$$

   (Notice the $1+\tan^2\theta$, which equals $\sec^2\theta$, under the square root; that is precisely the reason for the substitution we made)

$$= 5\int \sec\theta\sec^2\theta\, d\theta = \sqrt{5}\int \sec^3\theta\, d\theta,$$

   which now reduces to the integral in Fill-In #1.

4. $\displaystyle\int_1^\infty \frac{5}{x^3}\,dx = \underline{\quad \frac{5}{2} \quad}$.

We have

$$\begin{aligned}\int_1^\infty \frac{5}{x^3}\,dx &= \lim_{t\to\infty}\int_1^t \frac{5}{x^3}\,dx \\ &= \lim_{t\to\infty} -\frac{5}{2x^2}\Big|_1^t \\ &= \lim_{t\to\infty} -\frac{5}{2t^2}+\frac{5}{2} = \frac{5}{2}.\end{aligned}$$

5. If $\dfrac{x^2-3}{(x^2+1)(x-2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-2}$, then

(a) $A = \underline{\;\frac{4}{5}\;}$

(b) $B = \underline{\;\frac{8}{5}\;}$

(c) $C = \underline{\;\frac{1}{5}\;}$

We have $(Ax+B)(x-2)+C(x^2+1) = x^2-3$. Setting $x=2$ we get $5C=1$, so $C=\frac{1}{5}$. Now $A+C=1$, so $A=\frac{4}{5}$. Finally $-2A+B=0$, so $B=\frac{8}{5}$.

6. The polar curve $r=\cos\theta$ is symmetric about the **(** $\boxed{x\textbf{-axis}}$ | y**-axis** | **origin)** .

We have $\cos(-\theta)=\cos\theta$, so the curve is symmetric about the x-axis. However, $\cos(\pi-\theta)\neq\cos\theta$, so the curve is not symmetric about the y-axis. Also $-\cos\theta\neq\cos\theta$, so the curve is not symmetric about the origin.

Graph. *More accuracy = more points!*

Let C be the curve $\begin{array}{c} x=\cos t \\ y=\sin t\cos t \end{array}$.

(a) Eliminate the parameter to find a Cartesian equation of C.

$y^2=\sin^2 t\cos^2 t=(1-\cos^2 t)\cos^2 t=(1-x^2)x^2$. So we have $\boxed{y^2=(1-x^2)x^2 \text{ or } y=\pm x\sqrt{1-x^2}}$

(b) Find the point(s) on the curve where the tangent is vertical.

Since the curve is periodic with period 2π, we only need to consider t-values between 0 and 2π.

$\dfrac{dy}{dx}=\dfrac{y'}{x'}=\dfrac{-\sin^2 t+\cos^2 t}{-\sin t}=-\dfrac{\cos 2t}{\sin t}$.

The tangents will be vertical at those t-values for which $\dfrac{dy}{dx}$ is undefined (the denominator = 0). So set $\sin t=0$. The solutions are 0 and π. The points corresponding to these t-values are $\boxed{(1,0) \text{ and } (-1,0)}$

(c) Find the point(s) on the curve where the tangent is horizontal.

The tangents will be horizontal at those t-values for which $\frac{dy}{dx}$ is 0 (the numerator = 0).
Again, we only need to look for solutions on the interval $[0, 2\pi]$. So set $\cos 2t = 0$. The solutions are $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$. The points corresponding to these t-values are

$$\boxed{\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{1}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right), \text{ and } \left(\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)}$$

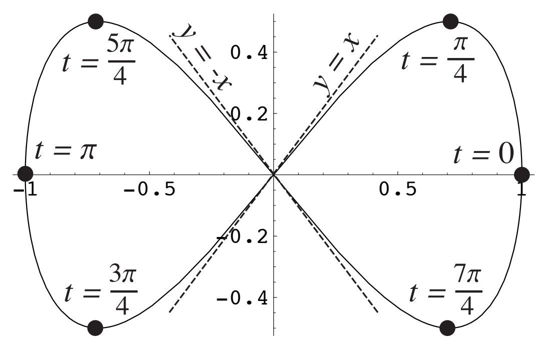
(d) Find equation(s) of the tangent(s) to C at the point $(0, 0)$.

Set $x = 0$ and $y = 0$:
$0 = \cos t \Rightarrow t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ are the only solutions in the interval $[0, 2\pi]$. Both of these t-values satisfy $0 = \sin t \cos t$, so there are two t-values corresponding to the point $(0, 0)$.
The slopes are as follows:

- $t = \frac{\pi}{2}$: $y'\left(\frac{\pi}{2}\right) = -\frac{\cos 2(\frac{\pi}{2})}{\sin \frac{\pi}{2}} = 1$
- $t = \frac{3\pi}{2}$: $y'\left(\frac{3\pi}{2}\right) = -\frac{\cos 2(\frac{3\pi}{2})}{\sin \frac{3\pi}{2}} = -1$

The y-intercept is $(0, 0)$ in both cases, so the equations are $y = x$ and $y = -x$.

(e) Sketch a graph of C, labeling the features found in parts (b)-(d).



Work and Answer. *You must show all relevant work to receive full credit.*

1. Evaluate the integral $\int \cos^2 x \sin^4 x \, dx$.

Since the powers of $\sin x$ and $\cos x$ are both even, we must use the half-angle and double-angle identities; we have

$$\begin{aligned}
\int \cos^2 x \sin^4 x \, dx &= \int (\cos x \sin x)^2 \sin^2 x \, dx \\
&= \int \left(\frac{1}{2}\sin 2x\right)^2 \left(\frac{1}{2}(1 - \cos 2x)\right) \, dx \\
&= \frac{1}{8}\int \sin^2 2x - \sin^2 2x \cos 2x \, dx \\
&= \frac{1}{8}\left[\int \left(\frac{1}{2}(1 - \cos 4x)\right) \, dx - \int \sin^2 2x \cos 2x \, dx\right]
\end{aligned}$$

(For the second integral, let $u = \sin 2x$ and proceed. I'll skip to the end of that. See me for details if you're not sure how I get there.)

$$= \boxed{\frac{1}{16}\left(x - \frac{1}{4}\sin 4x\right) - \frac{1}{48}\sin^3 2x + C}$$

2. Evaluate the integral $\int \tan^2 x \sec^4 x \, dx$.

   This one is actually much nicer than Work and Answer #1; since the power of $\sec x$ is even we can use the Pythagorean identity. We have

$$\begin{aligned}
\int \tan^2 x \sec^4 x \, dx &= \int \tan^2 x \sec^2 x \sec^2 x \, dx \\
&= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx \qquad \text{(Let } u = \tan x\text{)} \\
&= \int u^2 (u^2 + 1) \, du \\
&= \int u^4 + u^2 \, du \\
&= \boxed{\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C}
\end{aligned}$$

3. Evaluate the integral $\int \tan^3 x \sec^3 x \, dx$.

   This one is also nice because the power of $\tan x$ is odd. We have

$$\begin{aligned}
\int \tan^3 x \sec^3 x \, dx &= \int \tan^2 x \sec^2 x \cdot \sec x \tan x \, dx \\
&= \int (\sec^2 x - 1) \sec^2 x \cdot \sec x \tan x \, dx \qquad \text{(Let } u = \sec x\text{)} \\
&= \int (u^2 - 1) u^2 \, du \\
&= \int u^4 - u^2 \, du \\
&= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}
\end{aligned}$$

4. Evaluate the integral $\int \tan^2 x \sec x \, dx$.

   This one could get ugly because the power of $\sec x$ is odd *and* the power of $\tan x$ is even. We have

$$\begin{aligned}
\int \tan^2 x \sec x \, dx &= \int (\sec^2 x - 1) \sec x \, dx \\
&= \int \sec^3 x - \sec x \, dx. \qquad \text{Using Fill-In \#1:} \\
&= \left(\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x|\right) - \ln|\sec x + \tan x| + C \\
&= \boxed{\frac{1}{2}\left(\sec x \tan x - \ln|\sec x + \tan x|\right) + C}
\end{aligned}$$

5. Evaluate the integral $\int \cos 2x \sin 3x \; dx$.

We have, using one of the product identities,

$$\begin{aligned}\int \cos 2x \sin 3x \; dx &= \frac{1}{2}\int \sin x + \sin 5x \; dx \\ &= \frac{1}{2}\left(-\cos x - \frac{1}{5}\cos 5x\right) + C \\ &= \boxed{-\frac{1}{2}\left(\cos x + \frac{1}{5}\cos 5x\right) + C}\end{aligned}$$

6. Evaluate the integral $\int \sqrt{4-9x^2} \; dx$.

Here we must first see that $\int \sqrt{4-9x^2} \; dx = \int \sqrt{4-(3x)^2} \; dx = \frac{1}{3}\int \sqrt{4-u^2} \; du$ (where $u = 3x$), and then use the trigonometric substitution $u = 2\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

[Alternatively, if you feel confident you can substitute directly from the x's with $x = \frac{2}{3}\sin\theta$; then $dx = \frac{2}{3}\cos\theta \; d\theta$, and you'll get the same thing.]

In either case we end up with

$$\begin{aligned}\frac{1}{3}\int \sqrt{4-4\sin^2\theta} \cdot 2\cos\theta \; d\theta &= \frac{4}{3}\int \sqrt{1-\sin^2\theta}\cos\theta \; d\theta \\ &= \frac{4}{3}\int \sqrt{\cos^2\theta}\cos\theta \; d\theta \\ &= \frac{4}{3}\int \cos\theta\cos\theta \; d\theta \qquad \left(\text{since } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right) \\ &= \frac{4}{3}\int \cos^2\theta \; d\theta \\ &= \frac{4}{3}\cdot\frac{1}{2}\int 1 + \cos 2\theta \; d\theta \\ &= \frac{2}{3}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C \\ &= \frac{2}{3}(\theta + \sin\theta\cos\theta) + C\end{aligned}$$

Now use $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ to get $\theta = \sin^{-1}\left(\frac{u}{2}\right) = \sin^{-1}\left(\frac{3x}{2}\right)$ and trigonometry similar to what was done in class for §7.3 to get $\cos\theta = \frac{\sqrt{4-u^2}}{2} = \frac{\sqrt{4-9x^2}}{2}$. We have

$$\begin{aligned}&= \frac{2}{3}\left(\sin^{-1}\left(\frac{3x}{2}\right) + \frac{3x}{2}\cdot\frac{\sqrt{4-9x^2}}{2}\right) + C \\ &= \boxed{\frac{2}{3}\left(\sin^{-1}\left(\frac{3x}{2}\right) + \frac{3x\sqrt{4-9x^2}}{4}\right) + C}\end{aligned}$$

7. Evaluate the integral $\int \sqrt{4+9x^2}\,dx$.

Similar to above we use the substitution $x = \frac{2}{3}\tan\theta$; then $dx = \frac{2}{3}\sec^2\theta\,d\theta$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, and we get

$$\begin{aligned}
\frac{2}{3}\int \sqrt{4+4\sin^2\theta}\cdot\sec^2\theta\,d\theta &= \frac{4}{3}\int \sqrt{1+\tan^2\theta}\sec^2\theta\,d\theta \\
&= \frac{4}{3}\int \sqrt{\sec^2\theta}\sec^2\theta\,d\theta \\
&= \frac{4}{3}\int \sec^3\theta\,d\theta \qquad (\text{since } -\frac{\pi}{2} < \theta < \frac{\pi}{2}) \\
&= \frac{4}{3}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right) + C \qquad (\text{Using Fill-In \#1 again})
\end{aligned}$$

Again using trigonometry we get $\tan\theta = \frac{3x}{2}$ and $\sec\theta = \frac{\sqrt{4+9x^2}}{2}$. We have

$$\begin{aligned}
&= \frac{2}{3}\left(\frac{\sqrt{4+9x^2}}{2}\cdot\frac{3x}{2} + \ln\left|\frac{\sqrt{4+9x^2}}{2}+\frac{3x}{2}\right|\right) + C \\
&= \boxed{\frac{2}{3}\left(\frac{3x\sqrt{4+9x^2}}{4} + \ln\left|\frac{3x+\sqrt{4+9x^2}}{2}\right|\right) + C}
\end{aligned}$$

8. Evaluate the integral $\int \sqrt{9x^2-4}\,dx$.

Once more, with feeling! This time it's $x = \frac{2}{3}\sec\theta$ with $0 \le \theta < \frac{\pi}{2}$; then $dx = \frac{2}{3}\sec\theta\tan\theta\,d\theta$, and we get

$$\begin{aligned}
\frac{2}{3}\int \sqrt{4\sec^2\theta-4}\cdot\sec\theta\tan\theta\,d\theta &= \frac{4}{3}\int \sqrt{\sec^2\theta-1}\sec\theta\tan\theta\,d\theta \\
&= \frac{4}{3}\int \sqrt{\tan^2\theta}\sec\theta\tan\theta\,d\theta \\
&= \frac{4}{3}\int \tan^2\theta\sec\theta\,d\theta \qquad (\text{since } 0 \le \theta < \frac{\pi}{2}) \\
&= \frac{4}{3}\cdot\frac{1}{2}(\sec\theta\tan\theta - \ln|\sec\theta+\tan\theta|) + C
\end{aligned}$$

(using Work and Answer #4). Again using trigonometry we get $\sec\theta = \frac{3x}{2}$ and $\tan\theta = \frac{\sqrt{9x^2-4}}{2}$. We have

$$\begin{aligned}
&= \frac{2}{3}\left(\frac{3x}{2}\cdot\frac{\sqrt{9x^2-4}}{2} - \ln\left|\frac{3x}{2}+\frac{\sqrt{9x^2-4}}{2}\right|\right) + C \\
&= \boxed{\frac{2}{3}\left(\frac{3x\sqrt{9x^2-4}}{4} - \ln\left|\frac{3x+\sqrt{9x^2-4}}{2}\right|\right) + C}
\end{aligned}$$

9. Find the length of the curve $f(x) = \dfrac{e^x + e^{-x}}{2}$ from $x = 0$ to $x = 1$.

$f'(x) = \dfrac{e^x - e^{-x}}{2}$, so

$$1 + (f'(x))^2 = 1 + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$$
$$= \frac{1}{4}(e^{2x} + 2 + e^{-2x})$$
$$\frac{1}{4}(e^x + e^{-x})^2.$$

Therefore the length is

$$\begin{aligned} L &= \int_0^1 \sqrt{\frac{1}{4}(e^x + e^{-x})^2}\,dx \\ &= \frac{1}{2}\int_0^1 e^x + e^{-x}\,dx \\ &= \frac{1}{2}e^x - e^{-x}\Big|_0^1 \\ &= \frac{1}{2}\left(e - \frac{1}{e} - (1-1)\right) = \boxed{\frac{1}{2}\left(e - \frac{1}{e}\right)} \end{aligned}$$

10. Find the area of the surface formed by rotating the curve $f(x) = \dfrac{e^x + e^{-x}}{2}$ from $x = 0$ to $x = 1$ about the x-axis.

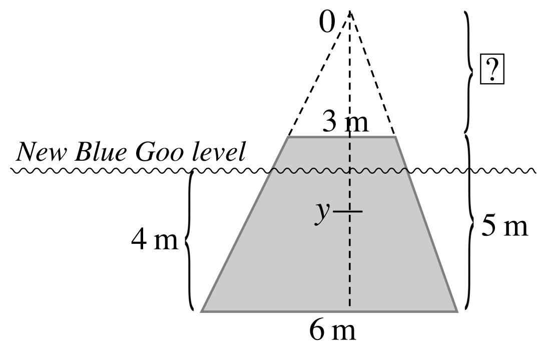
From Work and Answer #9, we have $\sqrt{1 + (f'(x))^2} = \frac{1}{2}(e^x + e^{-x})$. Therefore we have

$$\begin{aligned} SA &= 2\pi \cdot \frac{1}{2} \cdot \frac{1}{2}\int_0^1 (e^x + e^{-x})(e^x + e^{-x})\,dx \\ &= \frac{\pi}{2}\int_0^1 (e^x + e^{-x})^2\,dx \\ &= \frac{\pi}{2}\int_0^1 (e^{2x} + 2 + e^{-2x})\,dx \\ &= \frac{\pi}{2}\left(\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}\right)\Big|_0^1 \\ &= \frac{\pi}{2}\left(\frac{1}{2}e^2 + 2 - \frac{1}{2}e^{-2} - (\frac{1}{2} - \frac{1}{2})\right) \\ &= \boxed{\frac{\pi}{2}\left(\frac{e^2}{2} + 2 - \frac{1}{2e^{-2}}\right)} \end{aligned}$$

11. Find the hydrostatic force on the wall shown. The fluid is New Blue Goo (density 1500 kg/m^3).

There are several ways to do this problem. Please note that there is no principle of "similar trapezoids" like there is with similar triangles. We can, however, use similar triangles if we add a "top" to our trapezoid as shown.

I have chosen to make the top of the triangle my origin. The first thing we need to do is figure out what $\boxed{?}$ is. Using similar triangles, we have $\frac{\boxed{?}+5}{6} = \frac{\boxed{?}}{3}$, so $\boxed{?} = 5$. Now using similar triangles again we have $w(y) = \frac{3y}{\boxed{?}} = \frac{3y}{5}$ (I'm skipping some steps here). We also have $d(y) = y - 6$ (again using the top of the "triangle" as the origin).



Therefore the hydrostatic force is

$$\begin{aligned}
F &= 1500 \cdot 9.8 \int_6^{10} \frac{3y}{5}(y-6)\,dy = \frac{1500 \cdot 9.8 \cdot 3}{5} \int_6^{10} y(y-6)\,dy \\
&= 900 \cdot 9.8 \int_6^{10} y^2 - 6y\,dy = 900 \cdot 9.8 \left(\frac{1}{3}y^3 - 3y^2\right)\Big|_6^{10} \\
&= 900 \cdot 9.8 \left(\left(\frac{1000}{3} - 300\right) - \left(\frac{216}{3} - 108\right)\right) \\
&= 900 \cdot 9.8 \left(\frac{1000}{3} - 300 - 72 + 108\right) = \boxed{900 \cdot 9.8 \left(\frac{1000}{3} - 264\right) \text{ N}}
\end{aligned}$$

12. For the curve $\begin{array}{c} x = 1 + \tan t \\ y = \cos 2t \end{array}$, find $\frac{dy}{dx}$ in terms of t.

$\frac{dy}{dt} = -2\sin 2t$ and $\frac{dx}{dt} = \sec^2 t$, so $\frac{dy}{dx} = \frac{-2\sin 2t}{\sec^2 t} = \boxed{-4\sin t \cos^3 t}$

13. Find the point(s) on the polar curve $r = e^\theta$ where the tangent is
    (a) horizontal
    (b) vertical.

(a) The parametric equations for the curve are $\begin{array}{c} x = e^\theta \cos\theta \\ y = e^\theta \sin\theta \end{array}$. So we have

$$y' = e^\theta \cos\theta + e^\theta \sin\theta = e^\theta(\cos\theta + \sin\theta) \overset{\text{set}}{=} 0.$$

Since e^θ is never 0, we have $\cos\theta + \sin\theta = 0$ or $\tan\theta = -1$, which is true for angles θ of the form $\frac{3\pi}{4} + k\pi$, where k is any integer. So the set of points for which the tangent is horizontal is $\boxed{\left\{\left(e^{\frac{3\pi}{4}+k\pi}, \frac{3\pi}{4} + k\pi\right) \mid k \text{ is any integer}\right\}}$

(b) Similarly, $x' = e^\theta \cos\theta - e^\theta \sin\theta = e^\theta(\cos\theta - \sin\theta) \overset{\text{set}}{=} 0$ which gives $\cos\theta - \sin\theta = 0$ or $\tan\theta = 1$, which is true when θ is of the form $\frac{\pi}{4} + k\pi$, where k is any integer. So the set of points for which the tangent is horizontal is $\boxed{\left\{\left(e^{\frac{\pi}{4}+k\pi}, \frac{\pi}{4} + k\pi\right) \mid k \text{ is any integer}\right\}}$