Math 75A Practice Midterm II - Solutions
Ch. 5-7, §§8-A – 8-C (Ebersole), §§1.6-2.4 (Stewart), **W2**

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

Multiple Choice. *Circle the letter of the best answer.*

1. If the distance of a train from a station at time t minutes is $s(t) = 30 - t^2$ meters, then the average velocity of the train during the second minute is

(a) 6 meters per minute
(b) $\boxed{\text{3 meters per minute}}$
(c) 4 meters per minute
(d) 26 meters per minute

To find the average velocity (the average rate of change of the distance function $s(t)$ during the second minute (from $t = 1$ to $t = 2$) we find the slope of the secant line to the graph through the two points. First we need to find the two points:

$$s(1) = 30 - 1^2 = 29, \text{ so one of the points is } (1, 29).$$
$$s(2) = 30 - 2^2 = 26, \text{ so the other point is } (2, 26).$$

Therefore the average velocity is

$$\frac{29 - 26}{1 - 2} = \frac{3}{-1} = -3 \text{ meters per minute.}$$

The negative answer means the train is getting closer to the station at a rate of

$\boxed{\text{3 meters per minute}}$

2. The slope of the tangent line to the graph of $f(x) = |x + 2|$ at $x = -3$ is

(a) 1
(b) $\boxed{-1}$
(c) 0
(d) undefined.

$f(x) = \begin{cases} x + 2 & \text{if } x + 2 \geq 0 \\ -(x + 2) & \text{if } x + 2 < 0. \end{cases}$ In other words, $f(x)$ has slope 1 if $x+2 > 0$ and -1 if $x+2 < 0$.
Since $x = -3$ satisfies the second case $(-3 + 2 < 0)$, the slope there is -1.

3. If $H'(2) = \lim_{h \to 0} \dfrac{\frac{3}{\sqrt[4]{2+h}} - \frac{3}{\sqrt[4]{2}}}{h}$, then $H(t)$ could be

(a) $\boxed{\dfrac{3}{\sqrt[4]{t}}}$
(b) $-\dfrac{3}{\sqrt[4]{t}}$
(c) $-\dfrac{3}{\sqrt[4]{t^5}}$
(d) $12\sqrt[4]{t^3}$

According to the formula, $H'(2) = \lim_{h \to 0} \dfrac{H(2 + h) - H(2)}{h}$. We want this to be equal to $\lim_{h \to 0} \dfrac{\frac{3}{\sqrt[4]{2+h}} - \frac{3}{\sqrt[4]{2}}}{h}$.

Guess: try $H(t) = \dfrac{3}{\sqrt[4]{t}}$, since we see that $2+h$ is being plugged into that pattern in the formula we are given.

Check: Using our guess, we get

$$H(2+h) = \frac{3}{\sqrt[4]{2+h}}$$
$$H(2) = \frac{3}{\sqrt[4]{2}}.$$

This is exactly what we wanted to get. So the answer is $\boxed{H(t) = \dfrac{3}{\sqrt[4]{t}}}$.

4. $\dfrac{3}{5(\sqrt[4]{x+2})^3} + \dfrac{x^2}{3} - \sqrt{5x} =$

(a) $\frac{3}{5}(x+2)^{-3/4} + \frac{1}{3}x^{-2} - 5x^{1/2}$

(b) $\frac{3}{5}(x+2)^{-3/4} + \frac{1}{3}x^2 - \sqrt{5}x$

(c) $\boxed{\frac{3}{5}(x+2)^{-3/4} + \frac{1}{3}x^2 - \sqrt{5}x^{1/2}}$

(d) $\frac{3}{5}(x+2)^{-4/3} + \frac{1}{3}x^2 - \sqrt{5}x^{1/2}$

Using the rules of exponents, we get

$$\begin{aligned}\frac{3}{5(\sqrt[4]{x+2})^3} + \frac{x^2}{3} - \sqrt{5x} &= \frac{3}{5(x+2)^{3/4}} + \frac{1}{3}x^2 - \sqrt{5}\sqrt{x} \\ &= \frac{3}{5}(x+2)^{-3/4} + \frac{1}{3}x^2 - \sqrt{5}x^{1/2}.\end{aligned}$$

Use the following information to answer questions 5 and 6. Let $A(t)$ be the concentration of a certain drug in a patient's bloodstream, measured in g/m^3, t minutes after injection. Suppose $A'(30) = 2.4$ and $A'(90) = -1.3$.

5. $A'(30) = 2.4$ means

(a) After 30 minutes, the concentration of the drug in the patient's bloodstream is 2.4 g/m^3

(b) During the first 30 minutes after being injected, the concentration of the drug in the patient's bloodstream increased by an average of 2.4 g/m^3 per minute

(c) [After 30 minutes, the concentration of the drug in the patient's bloodstream is increasing at a rate of 2.4 g/m^3 per minute]

(d) After 2.4 minutes, the concentration of the drug in the patient's bloodstream has risen by 30 g/m^3

$A'(30)$ represents the rate of change of $A(t)$ at $t = 30$. So it is the rate at which the concentration of the drug is changing, measured in g/m^3 per minute, at $t = 30$ minutes after the patient was injected.

6. $A'(90)$ is a negative number, which means

   (a) After 90 minutes, there is a negative amount of the drug in the patient's bloodstream

   (b) $\boxed{\text{After 90 minutes, the concentration of the drug in the patient's bloodstream is decreasing}}$

   (c) 1.3 minutes before the injection, the concentration of the drug in the patient's bloodstream was 90 g/m^3

   (d) There is a mistake; $A'(90)$ cannot be a negative number

   A negative slope corresponds to a negative rate of change — in other words, a decrease.

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7. If $f(x) = 4x^7 - \dfrac{x^2}{5} + 2$, then $f'(x) =$

   (a) $28x^5 - \dfrac{2}{5}x$

   (b) $\boxed{28x^6 - \dfrac{2}{5}x}$

   (c) $4x^7 - \dfrac{1}{5}x^2 + 2$

   (d) $28x^6 - \dfrac{1}{5}x + 2$

   Since $\dfrac{x^2}{5} = \dfrac{1}{5}x^2$, using the power rule with the sum and difference rule we get the answer shown. Note that the derivative of a constant is 0, so the "+2" goes away in the answer.

8. If $g(x) = 6\sqrt[3]{x}$, then $g'(x) =$

   (a) $\boxed{\dfrac{2}{x^{2/3}}}$

   (b) $\dfrac{6}{x^{-1/3}}$

   (c) $6\sqrt[3]{1}$

   (d) $\dfrac{2}{\sqrt[3]{x}}$

   $g(x) = 6x^{1/3}$, so using the power rule we get $g'(x) = 2x^{-2/3} = \dfrac{2}{x^{2/3}}$.

9. <u>At $x = 3$</u> the graph of $f(x) = \dfrac{x-1}{x^2 - 4x + 3}$

   (a) is continuous

   (b) has a hole

   (c) $\boxed{\text{has a vertical asymptote}}$

   (d) has none of the above

   We have $f(x) = \dfrac{x-1}{(x-1)(x-3)}$, which is undefined at $x = 3$. Since the factor $x - 3$ does not cancel, we know there is a vertical asymptote at $x = 3$.

10. If $f(x) = \dfrac{|x+2|}{x+2}$ then $f'(-3) =$

    (a) $\boxed{0}$

    (b) -3

    (c) -1

    (d) -2

We have $f(x) = \begin{cases} \dfrac{x+2}{x+2} & \text{if } x+2>0 \\ -\dfrac{x+2}{x+2} & \text{if } x+2<0 \end{cases}$, which simplifies to $\begin{cases} 1 & \text{if } x>-2 \\ -1 & \text{if } x<-2 \end{cases}$. Therefore, at $x=-3$ $f(x)$ is a horizontal line (the line $y=-1$). Thus the slope (derivative) is equal to 0.

11. $\displaystyle\lim_{x\to\infty} \frac{3x^4-4x^2+2x-1}{5-x^4} =$

(a) $\dfrac{3}{5}$
(b) $\boxed{-3}$
(c) ∞
(d) 0

The degree of the numerator is equal to the degree of the denominator, so the limit at infinity is equal to the leading coefficient of the top over the leading coefficient of the bottom, or $\dfrac{3}{-1} = -3$. We can also use the "trick" of multiplying the top and bottom by 1 over the biggest power of x in the denominator; we get

$$\lim_{x\to\infty} \frac{3x^4-4x^2+2x-1}{5-x^4} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} = \lim_{x\to\infty} \frac{3-\frac{4}{x^2}+\frac{2}{x^3}-\frac{1}{x^4}}{\frac{5}{x^4}-1}$$
$$= \frac{3}{-1} = -3.$$

12. If $x<0$, then $\sqrt[6]{\dfrac{1}{x^{18}}} =$

(a) $\dfrac{1}{x^3}$
(b) $\boxed{-\dfrac{1}{x^3}}$
(c) $\dfrac{1}{x^{1/3}}$
(d) $-\dfrac{1}{x^{1/3}}$

This is the "really awful truth" about $x<0$. If we plug in $x=-1$, we see that $\sqrt[6]{\dfrac{1}{x^{18}}} \neq \dfrac{1}{x^3}$. But $\sqrt[6]{\dfrac{1}{x^{18}}} = -\dfrac{1}{x^3}$ does hold for $x<0$.

13. $-\dfrac{8\pi}{3}$ is an angle in Quadrant

(a) I
(b) II
(c) $\boxed{\text{III}}$
(d) IV

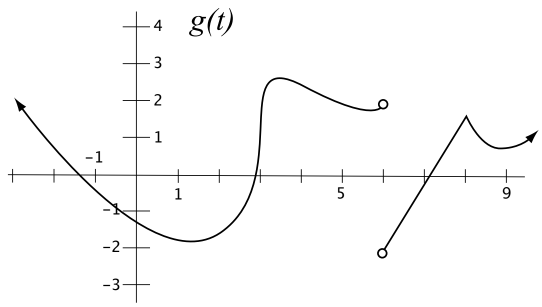
The terminal side of $-\dfrac{9\pi}{3} = -3\pi$ is pointing to the left, so proceeding in a clockwise direction (the negative angle direction) the angle $-\dfrac{8\pi}{3}$ comes just before it, in Quadrant III.

Fill-In.

1. For the graph of $g(t)$ shown at right, the value(s) of t at which $g'(t)$ is undefined is/are

   $\underline{\text{3, 6, and 8}}$.

   At $t = 3$ the graph has a vertical tangent, so the slope is undefined. At $t = 6$ $g(t)$ is not continuous, so $g'(6)$ cannot be defined. Finally, at $t = 8$ there is a corner, so the derivative is not defined there either.



2. $\displaystyle\lim_{x\to -1^+} \frac{x-5}{x+1} = \underline{-\infty}$

   $f(x) = \dfrac{x-5}{x+1}$ has a vertical asymptote at $x = -1$, so the limit as x approaches -1 from the right is either ∞ or $-\infty$. Plugging in $x = -0.99$ (a number just to the right of -1), we see that $f(x)$ comes out negative. So the answer to the limit is $-\infty$.

3. $\displaystyle\lim_{x\to 5} \frac{x-5}{x^2-12x+35} = \underline{-\frac{1}{2}}$

   $$\lim_{x\to 5} \frac{x-5}{x^2-12x+35} = \lim_{x\to 5} \frac{x-5}{(x-5)(x-7)} = \lim_{x\to 5} \frac{1}{x-7} = -\frac{1}{2}$$

4. $\displaystyle\lim_{x\to -\infty} \frac{3x^6-4x^5+x^2+3x}{\sqrt{5x^4-x^3-1}} = \underline{\infty}$

   Since the degree of the top is bigger than the degree of the bottom, we know the answer will be either ∞ or $-\infty$. The biggest power of x in the denominator is x^4. So we have

   $$\begin{aligned}\lim_{x\to -\infty} \frac{3x^6-4x^5+x^2+3x}{\sqrt{5x^4-x^3-1}} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} &= \lim_{x\to -\infty} \frac{3x^2-4x+\frac{1}{x^2}+\frac{3}{x^3}}{\sqrt{5}-\frac{1}{x}-\frac{1}{x^4}} \\ &= \lim_{x\to -\infty} \frac{3x^2-4x}{\sqrt{5}} = \infty.\end{aligned}$$

5. $\displaystyle\lim_{x\to \infty} \frac{2x+3}{4x^3-x+8} = \underline{0}$

   Since the degree of the bottom is bigger than the degree of the top, we know the answer is 0. You can also get this answer by multiplying the top and bottom by $\frac{1}{x^3}$.

6. $\cos\left(\dfrac{3\pi}{4}\right) = \underline{-\dfrac{\sqrt{2}}{2}}$

   The reference angle is $\dfrac{\pi}{4}$. $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$. $\dfrac{3\pi}{4}$ is in quadrant II, so $\cos\left(\dfrac{3\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$.

7. $\tan\left(\frac{11\pi}{6}\right) =$ $-\frac{\sqrt{3}}{3}$

The reference angle is $\frac{\pi}{6}$. $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. $\frac{11\pi}{6}$ is in quadrant IV, so $\tan\left(\frac{11\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

8. $\sec\left(\frac{17\pi}{3}\right) =$ 2

The reference angle is $\frac{\pi}{3}$. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, so $\sec\left(\frac{\pi}{3}\right) = 2$. $\frac{17\pi}{3}$ is in quadrant IV, so $\sec\left(\frac{17\pi}{3}\right) = 2$.

9. $\sin\left(-\frac{3\pi}{2}\right) =$ 1

The terminal side of the angle $-\frac{3\pi}{2}$ points straight up, so it intersects the unit circle at the point $(0, 1)$. $\sin\left(-\frac{3\pi}{2}\right)$ is the y-coordinate of this point, so the answer is 1.

10. If $\cos\theta = -\frac{1}{5}$ and θ is in quadrant II, then

(a) $\sin\theta = \frac{\sqrt{24}}{5}$

(b) $\tan\theta = -\sqrt{24}$

(c) $\sec\theta = -5$

(d) $\csc\theta = \frac{5}{\sqrt{24}}$

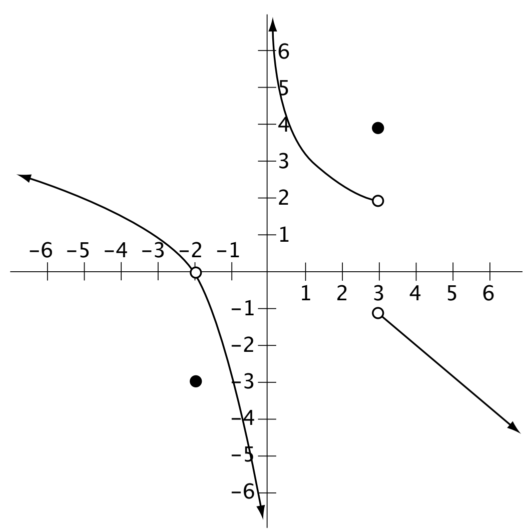
(e) $\cot\theta = -\frac{1}{\sqrt{24}}$

Graphs. *More accuracy = more points!*

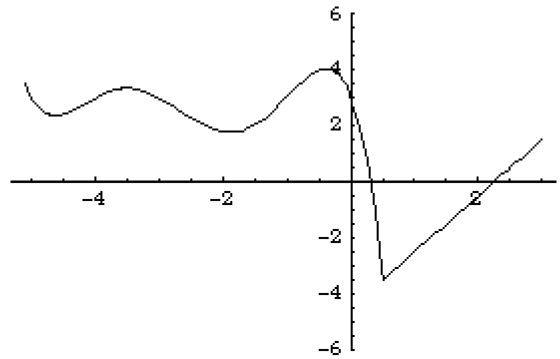
1. On the axes at right, sketch a graph of any function $f(x)$ satisfying all of the following:

- $\lim_{x\to 3^-} f(x) = 2$
- $\lim_{x\to 3^+} f(x) = -1$
- $f(3) = 4$
- $\lim_{x\to -2} f(x) = 0$
- $\lim_{x\to 0} f(x)$ does not exist

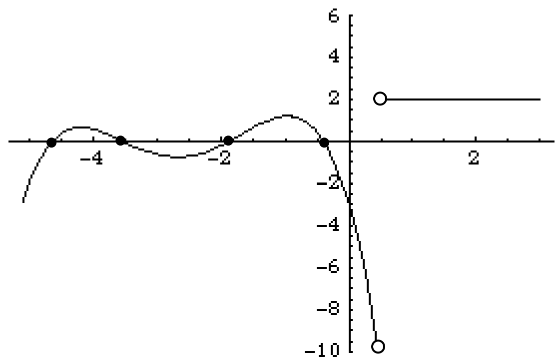
One possible graph is shown. There are many correct solutions.



2. For the graph of $f(x)$ shown at right, sketch a graph of $f'(x)$ on the axes below it.



The places where the graph has a horizontal tangent correspond to the zeros on the graph of the derivative (highlighted by dots on the x-axis). In between, the slope is either positive (above the x-axis) or negative (below the x-axis). The derivative of $f(x)$ is undefined at $x = 0.5$, since there is a corner there. Finally, the part of the graph that is a straight line has constant, positive slope approximately equal to 2. So the graph of the derivative shows a horizontal line at $y = 2$.



Work and Answer. *You must show all relevant work to receive full credit.*

1. Compute $\lim_{x\to 0^-} \dfrac{|x|-x}{x}$. If the limit does not exist, explain why.

$$\frac{|x|-x}{x} = \begin{cases} \dfrac{x-x}{x} & \text{if } x > 0 \\ \dfrac{-x-x}{x} & \text{if } x < 0 \end{cases} \quad \text{(we write $>$ instead of $\geq$ since $\frac{|x|-x}{x}$ is undefined at $x = 0$)}$$

$$= \begin{cases} \dfrac{0}{x} & \text{if } x > 0 \\ \dfrac{-2x}{x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } x > 0 \\ -2 & \text{if } x < 0. \end{cases}$$

Since we are only interested in the limit *from the left*, we need only consider the second case above. So $\lim_{x\to 0^-} \dfrac{|x|-x}{x} = \boxed{-2}$.

2. Compute $\lim_{x\to \infty} \dfrac{|x|-x}{x}$. If the limit does not exist, explain why.

This is similar to Work and Answer #1, except here we are taking the limit as x approaches ∞. Since the function has output 0 for all $x > 0$ (see the above solution to #1), we conclude that $\lim_{x\to\infty} \dfrac{|x|-x}{x} = \boxed{0}$

3. If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 m/s, then its height (in meters) after t seconds is $s(t) = 10t - 0.83t^2$.

(a) What is the velocity of the stone after 3 seconds?

To get the velocity, we take the derivative of the distance: $s'(t) = 10 - 1.66t$. Then $s'(3) = 10 - 1.66(3) = \boxed{5.02 \text{ m/s}}$.

(b) When does the stone reach its maximum height?

When the stone is at its maximum height, the velocity is 0. So we may get the answer by setting the velocity equal to 0 and solving for t:

$$10 - 1.66t \overset{\text{set}}{=} 0$$
$$1.66t = 10$$
$$t = \frac{10}{1.66} = \frac{1000}{166} = \boxed{\frac{500}{83} \approx 6 \text{ seconds}}$$

4. For the function $g(x) = \dfrac{2}{x-1}$, compute $g'(-1)$.

No shortcuts are allowed!

You can do this problem in two ways, but both of them have to use the formula $\lim\limits_{h\to 0} \ldots$ (or the other formula $\lim\limits_{x\to a} \ldots$).

You can use $a = -1$ right away; or you can compute the formula using a and then plug in -1 at the end.

Method 1. Plug in -1 first.

We get

$$g'(-1) = \lim_{h\to 0} \frac{g(-1+h) - g(-1)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2}{(-1+h)-1} - \frac{2}{-1-1}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2}{h-2} + 1}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2+(h-2)}{h-2}}{h}$$
$$= \lim_{h\to 0} \frac{2+h-2}{h(h-2)}$$
$$= \lim_{h\to 0} \frac{h}{h(h-2)}$$
$$= \lim_{h\to 0} \frac{1}{h-2}$$
$$= \frac{1}{0-2} = \boxed{-\frac{1}{2}}$$

Method 2. Get the formula, then plug in -1 at the end.

We get

$$g'(a) = \lim_{h\to 0} \frac{g(a+h) - g(a)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2}{(a+h)-1} - \frac{2}{a-1}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2(a-1)-2(a+h-1)}{(a+h-1)(a-1)}}{h}$$
$$= \lim_{h\to 0} \frac{2a - 2 - 2a - 2h + 2}{h(a+h-1)(a-1)}$$
$$= \lim_{h\to 0} \frac{-2h}{h(a+h-1)(a-1)}$$
$$= \lim_{h\to 0} \frac{-2}{(a+h-1)(a-1)}$$
$$= \frac{-2}{(a-1)(a-1)}$$
$$= \frac{-2}{(a-1)^2}.$$

Then plugging in $a = -1$ we get $g'(-1) = \dfrac{-2}{(-1-1)^2} = \dfrac{-2}{4} = \boxed{-\dfrac{1}{2}}$

Notice that we got the same answer both ways.

5. Find the value(s) of x at which the tangent line to the graph of $f(x) = \dfrac{2x}{5x^2+1}$ is horizontal.

The tangent line is horizontal at those places where the derivative is zero. So we take $f'(x)$ and set it equal to 0. Since $f(x)$ is a quotient, we must use the quotient rule. We have

$$f'(x) = \frac{(5x^2+1)2 - 2x(10x)}{(5x^2+1)^2} \overset{\text{set}}{=} 0$$

$$\frac{10x^2 + 2 - 20x^2}{(5x^2+1)^2} = 0$$

$$2 - 10x^2 = 0 \quad \text{(simplify the numerator and multiply both sides by the denominator)}$$

$$10x^2 = 2$$

$$x^2 = \frac{1}{5}$$

$$\boxed{x = \pm\sqrt{\frac{1}{5}} = \pm\frac{1}{\sqrt{5}}}$$

6. If $g(x) = \left(\sqrt[3]{x} - \dfrac{4}{x}\right)(3x-5)$, find $g'(8)$.

Since $g(x)$ is a product, we must use the product rule. First we rewrite $g(x)$ in a more "derivative-friendly" way. We have

$$g(x) = \left(x^{1/3} - 4x^{-1}\right)(3x-5),$$

so

$$\begin{aligned} g'(x) &= \left(\sqrt[3]{x} - \frac{4}{x}\right)3 + (3x-5)\left(\frac{1}{3}x^{-2/3} + 4x^{-2}\right) \\ &= 3\left(\sqrt[3]{x} - \frac{4}{x}\right) + (3x-5)\left(\frac{1}{3(\sqrt[3]{x})^2} + \frac{4}{x^2}\right). \end{aligned}$$

Now to get $g'(8)$ we just plug in $x = 8$:

$$\begin{aligned} g'(8) &= 3\left(\sqrt[3]{8} - \frac{4}{8}\right) + (3(8)-5)\left(\frac{1}{3(\sqrt[3]{8})^2} + \frac{4}{8^2}\right) \\ &= 3\left(2 - \frac{1}{2}\right) + 19\left(\frac{1}{12} + \frac{1}{16}\right) \\ &= \frac{9}{2} + 19\left(\frac{1}{12} + \frac{1}{16}\right) = \frac{24\cdot 9 + 19(4+3)}{48} = \boxed{\frac{349}{48}} \end{aligned}$$

You don't have to simplify your answer to get full credit, but it is good practice!

7. Find the <u>equation</u> of the tangent line to the graph of $h(x) = \sqrt{5x} + 1$ at $x = 4$.

To get the equation of a tangent line, first find the slope. Since we can rewrite $h(x)$ as $\sqrt{5}x^{1/2} + 1$, we can use the power rule to get $h'(x) = \frac{1}{2} \cdot \sqrt{5}x^{-1/2} = \frac{\sqrt{5}}{2\sqrt{x}}$. Therefore the slope at $x = 4$ is $h'(4) = \frac{\sqrt{5}}{2\sqrt{4}} = \frac{\sqrt{5}}{4}$.

Next we plug in $x = 4$ to the function to get the point of tangency (a point on the graph and also on the tangent line). $h(4) = \sqrt{20} + 1 = 2\sqrt{5} + 1$. So the point of tangency is $(4, 2\sqrt{5} + 1)$.

Now we can use the slope and the point of tangency to get the equation. We get

$$2\sqrt{5} + 1 = \frac{\sqrt{5}}{4} \cdot 4 + b, \qquad \text{so}$$
$$b = 2\sqrt{5} + 1 - \frac{\sqrt{5}}{4} \cdot 4$$
$$= 2\sqrt{5} + 1 - \sqrt{5} = \sqrt{5} + 1.$$

So the equation of the line is $\boxed{y = \frac{\sqrt{5}}{4}x + \sqrt{5} + 1}$.

8. For the function $f(x) = \frac{2x^2 - x - 3}{x^2 - 4x - 5}$,

   (a) Find the equation(s) of the vertical asymptote(s) of $f(x)$.

   (b) Find the equation(s) of the horizontal asymptote(s) of $f(x)$.

   (a) Factoring, we get $f(x) = \frac{(2x-3)(x+1)}{(x-5)(x+1)}$. $f(x)$ is undefined at $x = 5$ and $x = -1$, since those are the zeros of the denominator. But $x + 1$ cancels, so there is no vertical asymptote at $x = -1$. Therefore the only vertical asymptote is at $\boxed{x = 5}$

   (b) To find horizontal asymptotes, we take the limits at infinity; we have

$$\lim_{x\to\infty} \frac{2x^2 - x - 3}{x^2 - 4x - 5} \cdot \frac{1}{x^2} \frac{1}{x^2} = \lim_{x\to\infty} \frac{2 - \frac{1}{x} - \frac{3}{x^2}}{1 - \frac{4}{x} - \frac{5}{x^2}}$$
$$= \frac{2}{1} = 2.$$

   The limit of $f(x)$ as x approaches $-\infty$ is exactly the same; therefore the only horizontal asymptote is $\boxed{y = 2}$

9. Compute $\lim_{x\to\infty} \dfrac{\sqrt[4]{7x^{12} - 4x^3 + 5}}{2x - \sqrt{5}x^3}$. If the limit does not exist, explain why.

We have $\dfrac{1}{x^3} = \sqrt[4]{\dfrac{1}{x^{12}}}$ for $x > 0$, so

$$\lim_{x\to\infty} \frac{\sqrt[4]{7x^{12} - 4x^3 + 5}}{2x - \sqrt{5}x^3} \cdot \frac{1}{x^3}\frac{1}{x^3} = \lim_{x\to\infty} \frac{\sqrt[4]{7x^{12} - 4x^3 + 5}\sqrt[4]{\frac{1}{x^{12}}}}{\frac{2}{x^2} - \sqrt{5}}$$

$$= \lim_{x\to\infty} \frac{\sqrt[4]{7 - \frac{4}{x^9} + \frac{5}{x^{12}}}\sqrt[4]{\frac{1}{x^{12}}}}{\frac{2}{x^2} - \sqrt{5}}$$

$$= \boxed{\frac{\sqrt[4]{7}}{\sqrt{5}}}$$

10. Compute $\lim_{x\to-\infty} \dfrac{\sqrt[4]{7x^{12} - 4x^3 + 5}}{2x - \sqrt{5}x^3}$. If the limit does not exist, explain why.

This is similar to Work and Answer #9, above, except that here we are taking the limit of the function as x approaches $-\infty$. Since the function has a fourth root, the "really awful truth" applies. We have $\dfrac{1}{x^3} = -\sqrt[4]{\dfrac{1}{x^{12}}}$ for $x < 0$, so the answer comes out the opposite to that of #9. Repeat the steps above, but put in an extra minus sign, and you should get

$$\lim_{x\to-\infty} \frac{\sqrt[4]{7x^{12} - 4x^3 + 5}}{2x - \sqrt{5}x^3} = \boxed{-\frac{\sqrt[4]{7}}{\sqrt{5}}}$$