Math 75A Practice Midterm III
Ch. 8, 13-15 (Ebersole), §§2.3 (2) - 3.4 (Stewart)

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit on questions or parts of questions designated as* ***Work and Answer***.
3. **No calculators or notes are allowed on this exam.**
4. You have 65 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.* Your final answers should be boxed or circled.
7. Don't stress! I'm rooting for you!

True or False. Circle **T** if the statement is *always* true; otherwise circle **F**.

1. If $g(x) = 3x^4 \sin x$, then $g'(x) = 12x^3 \cos x$. **T** **F**
2. $\sec\theta \tan\theta = \dfrac{\sin\theta}{\cos^2\theta}$ for all angles θ. **T** **F**
3. $\sin(5t) = 5\sin t$ for all angles t. **T** **F**
4. $\tan\left(\dfrac{2\pi}{3}\right) = -\sqrt{3}$. **T** **F**
5. The only solution to the equation $\cos t = -1$ is $t = \pi$. **T** **F**

Multiple Choice. *Circle the letter of the best answer.*

1. The inverse of the function $f(x) = 3x^4 + 1$ is $f^{-1}(x) =$

(a) $f^{-1}(x) = \sqrt[4]{\dfrac{x-1}{3}}$

(b) $f^{-1}(x) = \sqrt[4]{\dfrac{x}{3}} - 1$

(c) $f^{-1}(x) = \sqrt[4]{\dfrac{x}{3} - 1}$

(d) none of these; $f(x)$ does not have an inverse

2. $\dfrac{8^t 16^3}{2^t} =$

(a) 2^{3t-12}
(b) 2^{2t+12}
(c) 2^{12-t}
(d) 2^{8t+3}

3. The inverse of the function $f(x) = 5x^3$ is $f^{-1}(x) =$

(a) $\dfrac{\sqrt[3]{x}}{5}$
(b) $5\sqrt[3]{x}$
(c) $\sqrt[3]{\dfrac{x}{5}}$
(d) $\dfrac{1}{5x^3}$

4. If $\ln(3x-2) = 4$, then $x =$

(a) $\dfrac{e^4+2}{3}$
(b) $\dfrac{\ln 4 + 2}{3}$
(c) $\dfrac{4+\ln 2}{\ln 3}$
(d) $e^{4/3}+2$

5. The inverse of the function $f(x) = 7^{3x-2}$ is $f^{-1}(x) =$

(a) $\dfrac{\log_7(x)+2}{3}$
(b) $\log_7\left(\dfrac{3x}{2}\right)$
(c) $3\log_7(x)+2$
(d) $(3x-2)^7$

6. The inverse of the function $f(x) = 3\ln(5x-2)$ is $f^{-1}(x) =$

(a) $e^{\frac{5x-2}{3}}$
(b) $\dfrac{1}{3}e^{5x-2}$
(c) $\dfrac{e^{2x-3}}{5}$
(d) $\dfrac{e^{x/3}+2}{5}$

7. If $f(x) = \tan x$, then $f'(x) =$

(a) $\sec^2 x$
(b) $\dfrac{\sin x}{\cos x}$
(c) $\dfrac{1}{\tan x}$
(d) $\sec x \tan x$

8. If $f(x) = x\tan x$, then $f'(x) =$

(a) $\sec^2 x$
(b) $x\sec^2 x$
(c) $x\sec^2 x + \tan x$
(d) $2x\sec^2$

9. If $f(x) = \tan(3\sqrt{x} + e^x)$, then $f'(x) =$

(a) $\sec^2(3\sqrt{x} + e^x)$

(b) $\sec^2(3\sqrt{x} + e^x)\dfrac{3}{2\sqrt{x}} + e^x$

(c) $\sec^2(3\sqrt{x} + e^x)\left(\dfrac{3}{2\sqrt{x}} + e^x\right)$

(d) $\sec^2(x)\left(\dfrac{3}{2\sqrt{x}} + e^x\right)$

10. If $f(x) = 4^x$, then $f'(x) =$

(a) $(\ln 4) \cdot 4^x$

(b) 4^x

(c) $x \cdot 4^{x-1}$

(d) $\ln(4^x)$

11. If $f(t) = (\tan t)e^{3t}$, then $f'(t) =$

(a) $3e^{3t} \sec^2 t$

(b) $3e^{3t} \tan t + e^{3t} \sec^2 t$

(c) $\tan^3 t \sec^2(e^t)$

(d) $e^{3t} \tan t + 3e^{3t} \sec^2 t$

12. If $f(x) = \ln(\sin^7 x)$, then $f'(x) =$

(a) $7 \cot x$

(b) $7 \ln(\sin x)$

(c) $\dfrac{1}{\sin^7 x}$

(d) $\dfrac{7 \sin^6 x \cos x}{x}$

13. If $f(x) = \ln((2x-1)^{35}(5x^6+7)^{10})$, then $f'(x) =$

(a) $\dfrac{70}{2x-1} + \dfrac{300x^5}{5x^6+7}$

(b) $\dfrac{1}{(2x-1)^{35}(5x^6+7)^{10}}$

(c) $\dfrac{35}{2x-1} + \dfrac{10}{5x^6+7}$

(d) $\dfrac{1}{(2x-1)^{35}} + \dfrac{1}{(5x^6+7)^{10}}$

14. If $f(x) = \dfrac{\ln x}{x^3}$, then $f'(x) =$

(a) $\dfrac{1}{3x^3}$

(b) $3x^2 \cdot \dfrac{1}{x} + x^3 \ln x$

(c) $\dfrac{(1 - 3\ln x}{x^4}$

(d) $\dfrac{3x^2}{\ln x}$

15. If \$1000 is invested at 2% interest compounded annually, the balance after 10 years is

(a) $1000e^{1.02}$

(b) $2000e^{10}$

(c) $1000 \cdot 2^{10}$

(d) $1000(1.02)^{10}$

Fill-In. *Fill in the correct integer (positive, negative, or 0 whole number).*

1. $\log_2\left(\dfrac{1}{32}\right) =$ _______

2. $\log(100) =$ _______

3. $e^{4\ln 10} =$ _______

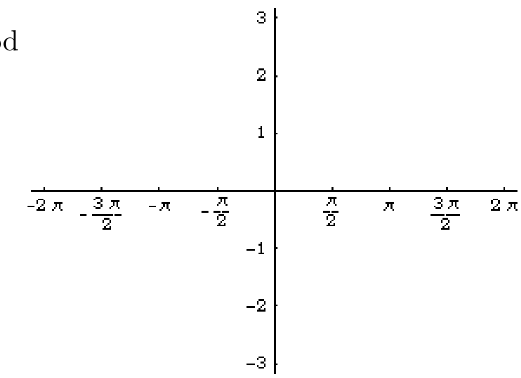
4. $\log_4(12) - \log_4(3) =$ _______

5. $3^{2\log_9(2007)} =$ _______

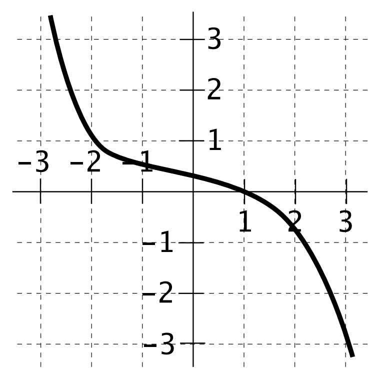
6. $\log_{1/4}(64) =$ _______

Graphs. *More accuracy = more points!*

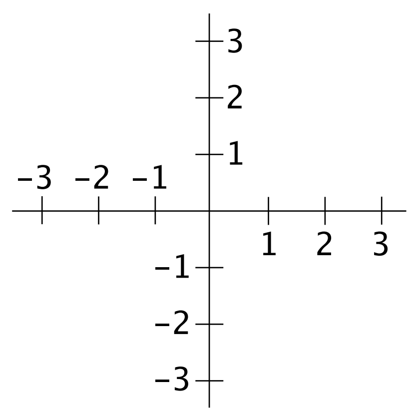
1. On the axes at right, sketch a graph of at least one period of the function $f(t) = \dfrac{3}{2}\sin(2t - \pi)$.



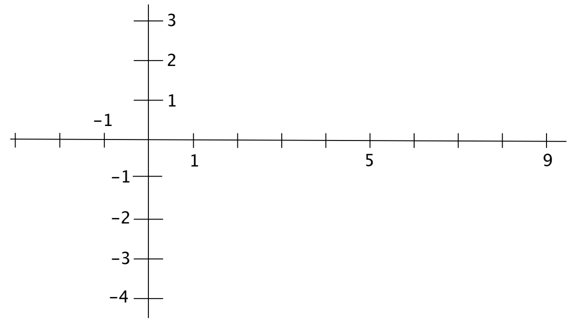
2. For the graph of $f(x)$ shown at right, sketch a graph of $f^{-1}(x)$ on the same axes.



3. On the axes below, sketch a graph of $f(x) = 2^{x+1} - 2$.
   Label at least two points on the curve.



4. On the axes below, sketch a graph of $f(x) = \log_2(x+1) - 3$.
   Label at least two points on the curve.



Work and Answer. *You must show all relevant work to receive full credit.*

1. Find the derivative of the function $f(x) = \dfrac{\sqrt{\cos x}}{e^{x^4+1}}$.

2. Find the inverse of the function $f(x) = \dfrac{3}{2x-5}$.

3. Simplify the expression $\log_3\left(\dfrac{27x^4}{3^{y+2}}\right)$.

4. A population of a certain strain of bacteria doubles every 24 hours. How long does it take for the population to triple?

5. The half-life of ^{237}U (an isotope of uranium) has a half-life of 6.75 days. How long does it take for a 100g sample to decay to 10g?

Some kind of **BONUS.**