Math 111 Practice Final
Ch. 0-9

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your steps thoroughly and unambiguously to receive full credit.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.*
7. Don't stress! I'm rooting for you!

This test is comprehensive. Review all terms, notations, and types of proofs in chapters 0-9. In particular, make sure to understand

- Communicating mathematics properly (chapter 0)
- Sets, including sets of numbers and their notations, subsets, set operations (union, intersection, difference, complement, product), and their notations and fundamental properties, Venn diagram, partitions (chapter 1, section 4.5)
- Propositions, propositional functions, logical operations (and, or, not, implication, biconditional) and their notations, truth tables, tautologies, contradictions, logical equivalence and its fundamental properties, quantifiers (universal and existential) (chapter 2)
- Types of proofs: trivial, vacuous, by cases, direct, by contrapositive, by contradiction (chapters 3-5)
- Definitions and properties of divisibility and congruences (sections 4.1-4.2)
- Various proof techniques for statements involving: sets, integers, rational/irrational numbers, positive/negative numbers, any real numbers, absolute value (chapters 4-5)
- Proof of the irrationality of $\sqrt{2}$ (section 5.5)

- Testing, proving and disproving (quantified) statements (including: when an example/ counterexample is sufficient? When a general proof is required?) (chapter 6)
- Relations and their properties (reflexive, symmetric, transitive), equivalence relations, equivalence classes, partitions (chapter 7)
- Functions and their properties (injective, surjective, bijective), composition of functions (chapter 8)
- Principle of Mathematical Induction (chapter 9)

1. Prove or disprove the following statement:
   Let A, B, and C be sets. Then $(A \cup B) - C = (A - C) \cup (B - C)$.
2. Determine whether the compound propositions $(P \vee Q) \Rightarrow (P \wedge Q)$ and $P \Leftrightarrow Q$ are logically equivalent.
3. Let $n \in \mathbb{Z}$. Prove that if $3n^2 + 4n + 2$ is even, then n is even.
4. Prove or disprove the following statement:
   For any $a \in \mathbb{Z}$, the number $a^3 + a + 100$ is positive.
5. Consider the relation R dened on $\mathbb{Z}$ by aRb iff $ab \leq 0$. Determine whether R is
   (a) reflexive
   (b) symmetric
   (c) transitive
   (d) an equivalence relation.
6. Consider the function $f\colon \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = \begin{cases} x & \text{if } x \text{ is even} \\ 2x & \text{if } x \text{ is odd} \end{cases}$
   Determine whether f is
   (a) one-to-one
   (b) onto
   (c) bijective.
7. Prove that the number 111 cannot be written as the sum of four integers, two of which are even and two of which are odd.
8. Prove that $7 \mid (3^{2n} - 2^n)$ for every nonnegative integer n.

Some kind of **BONUS.**
Possible question: Give an example of a bijective function $f\colon \mathbb{Z} \to \mathbb{N}$ and find its inverse.