Math 111 Practice Midterm I
Ch. 1, 2

DISCLAIMER. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your work thoroughly and unambiguously to receive full credit.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.*
7. Don't stress! I'm rooting for you!

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1. Review all terms and notations from chapters 0-2.

2. Let $U = \{x \in \mathbb{Z} \mid 0 \leq x \leq 10\}$ be the universal set, $A = \{x \in U \mid x \text{ is even}\}$, $B = \{1, 2, 3, 4, 5\}$.
   (a) Draw a Venn diagram that illustrates the above sets.
   (b) Determine (i.e. list all the elements of) the following sets: $A \cap B$, $\overline{A}$, $A \cup \overline{B}$.
   (c) How many elements does $A \times B$ have?
   (d) List three of the elements of $A \times B$.

3. Let $A = \{1\}$, $B = \{2\}$, $C = \{\{3\}\}$, $D = \{1, \{2\}, \{1, 2, 3\}\}$.
   (a) Which of the following statements are true: $A \in D$, $A \subseteq D$, $B \in D$, $B \subseteq D$, $C \in D$, $C \subseteq D$, $\emptyset \in D$, $\emptyset \subseteq D$?
   (b) What are the cardinalities of these four sets?

4. For each $n \in \mathbb{N}$ let $A_n = \left[\frac{1}{n}\ ,\ \frac{n+1}{n}\right)$. Determine $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$. (No formal proof is required, but please provide an explanation of your answer; a picture might be helpful.)

5. Let P and Q be statements.

   (a) Show that $P \iff Q$ and $(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$ are logically equivalent.

   (b) The compound statement $(P \iff Q) \iff ((P \wedge Q) \vee ((\sim P) \wedge (\sim Q)))$ is an example of a _____.

   (c) The compound statement $(P \iff Q) \iff \sim ((P \wedge Q) \vee ((\sim P) \wedge (\sim Q)))$ is an example of a _____.

6. Let

$$P(x) : x \text{ is wearing shoes.}$$
$$Q(x) : x \text{ has an umbrella.}$$
$$R(x) : x \text{ walks to class.}$$

   where x belongs to the set $S = \{\text{students at Fresno State}\}$.

   (a) Write the following statements in words:

   - $(P(\text{Jasper}) \wedge Q(\text{Jasper})) \Rightarrow R(\text{Jasper})$.
   - $\sim Q(\text{Cameron}) \Rightarrow \sim R(\text{Cameron})$.
   - $R(\text{Dirk}) \iff P(\text{Yolanda})$.
   - $\forall x, P(x) \wedge Q(x)$.
   - $\exists x, \sim (R(x) \Rightarrow P(x))$.

   (b) Determine the negations of the above statements. Write them in words and in symbols.

Some kind of **BONUS.**