

Math 111 Practice Midterm II

Ch. 3-5

DISCLAIMERS.

1. This collection of practice problems is *not* guaranteed to be identical, in length or content, to the actual exam. You may expect to see problems on the test that are not exactly like problems you have seen before.
2. This test is primarily on chapters 3-5. However, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0-6.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking.
2. *You must explain your steps thoroughly and unambiguously to receive full credit.*
3. **No calculators or notes are allowed on this exam.**
4. You have 50 minutes to complete your test, unless announced otherwise. Do not spend too long on any one problem. You do not have to do the problems in order. Do the easy ones first. Do not attempt the bonus question until you have completed the rest of the test. Before turning in your test, please make sure you have answered and double-checked all the questions.
5. If you need scratch paper, please raise your hand. You may not use your own paper. When you have finished your exam, please turn in any scratch paper you use.
6. Write your solutions in the space provided for each problem, or provide specific instructions as to where your work is to be found. *Make it clear what you want and don't want graded.*
7. Don't stress! I'm rooting for you!

For each problem, prove the statement. Indicate what type of proof (direct, by contrapositive, or by contradiction) you are using.

1. Let x be a real number.
 - (a) If $x > -7$, then $-5 - x^2 < 0$.

 - (b) If $|x| = 5$, then $x^2 + x + 1 > 20$.

 - (c) If $2x > x^2 + x^3$, then $x < 1$.

2. Let n and m be integers.

(a) If $3n^2 + 5n$ is odd, then $n \geq 10$.

(b) If n is even, then $3n^2 - 2n - 5$ is odd.

(c) If $7n^2 + 4$ is even, then n is even.

(d) If $n - 5m$ is odd, then n and m are of opposite parity.

(e) If $5 \mid (n - 1)$, then $5 \mid (n^3 + n - 2)$.

(f) $3 \mid mn$ if and only if $3 \mid m$ or $3 \mid n$.

3. The number $\log_3 2$ is irrational.

4. The product of a nonzero rational number and an irrational number is irrational.

5. Let A and B be sets. Then $A \cap B = \emptyset$ if and only if $(A \times B) \cap (B \times A) = \emptyset$.

Some kind of **BONUS**.