Math 75A Bonus Assignment – Derivatives of Logarithmic Functions Fall 2008

Solutions

Recall that the derivative of $f(x) = \ln x$ is $f'(x) = \frac{1}{x}$. Now try these:

For each function, fill in the derivative.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$3\ln x$	$\frac{3}{x}$	$\ln(3x^2+1)$	$\frac{1}{3x^2+1} \cdot 6x = \frac{6x}{3x^3+1}$
$x^2 \ln x$	$x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$ $= x + 2x\ln x$	$\ln\left((8x^5-3)\sin x\right)$	$\frac{(8x^5-3)\cos x + \sin x \cdot 40x^4}{(8x^5-3)\sin x}$

Do you see an easier way to do the last one above? We can use a logarithm law to rewrite the original function as

$$f(x) = \ln(8x^5-3) + \ln(\sin x)$$

and now the derivative should be much easier! Try it again using the above equation:

$$f'(x) = \frac{40x^4}{8x^5-3} + \frac{\cos x}{\sin x}$$

Are your two answers equal to each other (after some algebra)?

Yes. If we get a common denominator for the latter formula, we will get

$$\frac{40x^4}{8x^5-3} + \frac{\cos x}{\sin x} = \frac{40x^4 \sin x + (8x^5-3)\cos x}{(8x^5-3)\sin x}.$$

Now try these: for each function, do the following:

(a) Use logarithm laws to rewrite the function so that the terms are logarithms that are as simple as possible.

(b) Find the derivative of the function.

1. $f(x) = \ln\left(\frac{x^5 - e^x}{7x+1}\right)$

   (a) $f(x) = \ln(x^5 - e^x) - \ln(7x+1)$

   (b) $f'(x) = \frac{5x^4 - e^x}{x^5 - e^x} - \frac{7}{7x-1}$

2. $g(x) = \ln\left((x^6 + 6^x)^4\right)$

   (a) $g(x) = 4\ln(x^6 + 6^x)$

   (b) $g'(x) = 4 \cdot \dfrac{6x^5 + (\ln 6)\cdot 6^x}{x^6 + 6^x}$

3. $h(x) = \ln\left((4x^2 - \sqrt{x} + 7)^3(3e^x - 2)^5\right)$

   (a) $h(x) = 3\ln(4x^2 - \sqrt{x} + 7) + 5\ln(3e^x - 2)$

   (b) $h'(x) = 3 \cdot \dfrac{8x - \frac{1}{2}x^{-1/2}}{4x^2 - \sqrt{x} + 7} + 5 \cdot \dfrac{3e^x}{3e^x - 2}$

4. $k(x) = \ln\left(\dfrac{\sqrt[3]{9x^4 - 10^{4x-3}}}{(5x^2+3)^9(12\sqrt{x-2})^2}\right)$

   (a) $$\begin{aligned} k(x) &= \tfrac{1}{3}\ln(9x^4 - 10^{4x-3}) - \left(9\ln(5x^2+3) + 2\ln(12\sqrt{x-2})\right) \\ &= \tfrac{1}{3}\ln(9x^4 - 10^{4x-3}) - 9\ln(5x^2+3) - 2(\ln(12) + \ln(\sqrt{x-2})) \\ &= \tfrac{1}{3}\ln(9x^4 - 10^{4x-3}) - 9\ln(5x^2+3) - 2\ln(12) - 2\cdot\tfrac{1}{2}\ln(x-2) \end{aligned}$$

   (b) $$\begin{aligned} k'(x) &= \frac{1}{3}\cdot\frac{36x^3 - (\ln 10)\cdot 10^{4x-3}\cdot 4}{9x^4 - 10^{4x-3}} - 9\cdot\frac{10x}{5x^2+3} - 0 - \frac{1}{x-2} \\ &= \frac{36x^3 - 4\ln 10\cdot 10^{4x-3}}{3(9x^4 - 10^{4x-3})} - \frac{90x}{5x^2+3} - \frac{1}{x-2} \end{aligned}$$