Math 75B Worksheet
Higher Derivatives and Related Rates

1. If $f(t) = \sin(2t)$,

   (a) Find $f'(t)$.

   (b) Find $f''(t)$.

   (c) Find $f^{(4)}(t)$.

   (d) Find $f^{(6)}(t)$.

   (e) Find $f^{(8)}(t)$.

   (f) Find $f^{(100)}(t)$.

2. The surface area of a spherical balloon of radius r cm is $S = 4\pi r^2$ (in cm^2). As the balloon is inflated, both the radius and the surface area change with time t (in seconds).

   (a) Write symbols that represent the rate of change of the radius.
   (It might look like $\frac{d\,?}{dt}$.)

   (b) Write symbols that represent the rate of change of the surface area.

   (c) Differentiate the equation $S = 4\pi r^2$ with respect to t. *Your answer should contain the symbols you wrote in parts (a) and (b).*

   (d) If the radius is increasing at a constant rate of 3 cm/s, find the rate at which the surface area is increasing at the moment the radius is 10 cm.

over for more fun!

3. *Read the following problem carefully and then use the hints below to solve the problem.*

Mary and Sheila say goodbye, get in their cars and drive off. Mary heads east at 36 miles/hour, and Sheila heads north at 48 miles/hour. At what rate is the distance between them increasing after 20 minutes ($= \frac{1}{3}$ hour)?

Hints:

(a) As in the previous problem, the goal of the solution is to **write an equation** involving all the quantities that are *changing* in the problem — such as Mary's distance from the starting point, the distance between the two cars, etc. — and **take the derivative** with respect to t to see how the rates of change are related. That's why these problems are called *related rates* problems.

Important. Do not attempt to plug in $t = \frac{1}{3}$ anywhere until you have done all of the above. We need a *general* formula for how the rates are related before we can plug in information about a *specific* time.

(b) To help you get an equation, draw a picture of the situation described in the problem. It should probably look like a right triangle where one of the vertices is the starting point of the two cars. Then label your picture with some variables. For example, you could label Mary's distance from the starting point with "m." You will want to give names to all the distances described in the problem.

(c) The Pythagorean Theorem will be helpful in writing your equation.

(d) Always take the derivative of your equation with respect to t.

(e) Decide where to plug in 36 and 48 from the problem. Are these distances? Velocities?

(f) How far has Mary gone in 20 minutes? What about Sheila? How can you use the original equation to determine the distance between them after 20 minutes?

(g) What quantity are we trying to solve for in this problem? Which symbol represents that quantity?