Math 152 Practice Problems for Midterm I
§§1.1-2.1

DISCLAIMER. This collection of practice problems is not guaranteed to be identical, in length, format or content, to the actual exam. The exam will cover sections 1.1-1.6 and 2.1.

On the actual exam you will have more room to work the problems. You will see directions similar to these:

> 1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking. In problems designated **W** you must justify all steps to receive full credit.
> 2. Please write your solutions neatly in the space provided, or provide clear directions as to where your work is to be found. If you do problems on scratch paper, please be sure to label problems and indicate clearly what is to be graded.
> 3. Don't stress! I'm rooting for you!

If I were you, I would:

- Know all of the definitions mentioned in class and in the sections of the book, and know examples.
- Know all of the theorems mentioned in class and in the sections of the book, and know examples relating to them.
- Go over all of the homework problems, even "redoing" them on WeBWorK in order to practice.
- Do as many of the Warmup/Study/Enrichment problems from the Homework List as you can.
- Practice proving things such as exercises **Section 1.5** #48-54, **Section 1.6** #20-22 and **Section 2.1** #9, 10.

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1. **(W)** Find invertible $n \times n$ matrices A and B (for your choice of $n \geq 2$) such that

   (a) $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$
   (b) $(A + B)(A - B) \neq A^2 - B^2$
   (c) $A + I_n$ is not invertible
   (d) $5A$ is invertible
   (e) $ABA^{-1} = B$

2. **(W)** Find the matrix A of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -8 \end{bmatrix} x_1 + \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} x_2.$$

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

3. **(W)** Give an example of a matrix for each of the following transformations of $\mathbb{R}^2$:

   A. Identity transformation

   B. Contraction by a factor of 2

   C. Reflection in the x-axis

   D. Rotation through an angle of $60°$ in the counterclockwise direction

   E. Projection onto the x-axis

   F. Reflection in the y-axis

4. **(W)** For each row in the table below, find an example of a system that has the properties given in the row. If it is not possible, explain why not.

Equations	Unknowns	Solutions	Homogeneous
2	2	Unique	No
4	2	Infinite	Yes
4	2	None	No
2	3	Infinite	No
2	3	Unique	No
3	3	Infinite	Yes
3	3	None	No
3	3	None	Yes

5. **(W)** If A is an $n \times n$ invertible matrix, prove that $(A^{-1})^T = (A^T)^{-1}$.

6. **(W)** If A and B are $n \times n$ invertible matrices, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

7. **(W)** If A is a symmetric matrix, prove that A is square.

8. **(W)** Prove that if a matrix A is upper and lower triangular, then A is diagonal.

9. **(W)** Give examples:

   - $AB = AC$ but $B \neq C$.
   - A singular (i.e. non-invertible) square matrix.
   - $AB \neq BA$.
   - $AB = 0$ with $A \neq 0$ or $B \neq 0$.
   - A and B are row equivalent but not equal.

10. **(W)** For the matrix

$$A = \begin{bmatrix} 3 & 0 & 4 \\ 2 & 1 & 5 \\ 1 & -1 & 0 \end{bmatrix}$$

use Gaussian elimination to reduce A to row echelon form. *Be sure to note which elementary row operations you are performing in each step.*

11. Decide whether the following matrices are in row echelon form (RE), reduced row echelon form (RRE), or neither (N) and circle the best answer.

RE **RRE** **N**

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

RE **RRE** **N**

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

RE **RRE** **N**

$$\begin{bmatrix} 1 & 3 & 0 & 1 & -5 \\ 0 & 1 & 0 & 4 & 9 \\ 0 & 0 & -1 & 3 & 8 \\ 0 & 0 & 0 & 12 & -4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

RE **RRE** **N**

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

RE **RRE** **N**

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$