**Math 152 Final Exam** $/_{100}$ Name: ____________________

Chapters 1-7

> **Note.** Here is a draft of the final exam which I was intending to give you as practice. While it is not finalized, it is pretty close to what I intend to give. Unfortunately, my dog chewed it up before I could give it to you! So there may be some parts that are hard to read. I hope you can benefit from it anyway. Good luck!

Things to remember:

1. Please read directions carefully. Raise your hand if you are not sure what a problem is asking. In problems designated **W** you must justify all steps to receive full credit.
2. Please write your solutions neatly in the space provided, or provide clear directions as to where your work is to be found. If you do problems on scratch paper, please be sure to label problems and indicate clearly what is to be graded.
3. Don't stress! I'm rooting for you!

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The following formulas may be useful:

$$\mathbf{v}_i = \mathbf{u}_i - \frac{\mathbf{v}_1 \cdot \mathbf{u}_i}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \frac{\mathbf{v}_2 \cdot \mathbf{u}_i}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2 - \ldots - \frac{\mathbf{v}_{i-1} \cdot \mathbf{u}_i}{\mathbf{v}_{i-1} \cdot \mathbf{v}_{i-1}}\mathbf{v}_{i-1}$$

$$\mathbf{x}^T A\mathbf{x} + B\mathbf{x} + f = 0$$

1. (5 points) **(W)** Solve the following system of equations. Express your solution(s) as a vector or set of vectors.

   **(CHOMP)**

2. (5 points) **Multiple Choice.** *Circle the letter of the best answer.*

   The coordinate vector of **(CHOMP)** relative to the ordered basis **(SLOBBER)** is

   **(CHOMP)**

3. (6 points) If $A =$ **(CHOMP)**, then

   (A) $\det(A) =$ ______

   (B) $\det(A^T) =$ ______

   (C) $\det(A^{-1}) =$ ______

4. (5 points) **Multiple Choice.** *Circle the letter of the best answer.* A basis for the solution space of the homogeneous system **(CHOMP)** is

   **(GNAW)**

5. (12 points) **Multiple Choice.** *Circle the letter of the best answer.* The reduced row-echelon forms of the augmented matrices of four systems of equations are given below. How many solutions does each system have?

   A. **(CHOMP)**

   (a) Unique solution
   (b) No solutions
   (c) Infinitely many solutions
   (d) None of the above

   B. **(CHEW)**

   (a) Unique solution
   (b) Infinitely many solutions
   (c) No solutions
   (d) None of the above

   C. **(GNAW)**

   (a) Infinitely many solutions
   (b) Unique solution
   (c) No solutions
   (d) None of the above

   D. **(SLOBBER)**

   (a) No solutions
   (b) Unique solution
   (c) Infinitely many solutions
   (d) None of the above

6. (6 points) (**W**) Consider the set $W =$ **(CHOMP)**. Prove that $W$ is a subspace of $M_{23}$ (the vector space of all $2 \times 3$ matrices with real entries).

   *You do not need to prove that $M_{23}$ is a vector space.*

7. (5 points) (**W**) Prove or disprove: the set $S =$ **(CHOMP)** is a linearly independent subset of $\mathbb{R}^4$.

8. (5 points) (**W**) Does the set $T =$ **(CHOMP)** span the vector space

$$P_3 = \{\text{polynomials of degree} \leq 3\}?$$

   Why or why not?

9. (6 points)

(A) Complete the table as follows: for each set $V$, determine whether or not $V$ is a vector space with the operations of addition $\oplus$ and scalar multiplication $\odot$ given. Then give an example of an element of $V$, as in the sample.

| $V$ | $\oplus$ | $\odot$ | Vector space? | Element of $V$ |
|---|---|---|---|---|
| $\mathbb{R}^2$ | $\mathbf{x} \oplus \mathbf{y} = \mathbf{x} + \mathbf{y}$ | $c \odot \mathbf{x} = c\mathbf{x}$ | YES | $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ |
| **(CHOMP)** | **(SLOBBER)** | **(CHEW)** | | |
| **(GNAW)** | **(RIP)** | **(GULP)** | | |

(B) If either of the sets in (A) is not a vector space, give at least one property of a vector space that fails to hold for $(V, \oplus, \odot)$ and show using a counterexample that the property fails.

10. (5 points) **Multiple Choice.** *Circle the letter of the best answer.*

If $W$ is the subspace of $\mathbb{R}^3$ spanned by **(CHOMP)**, then the dimension of $W$ is

(a) 0

(b) 1

(c) 2

(d) 3

11. (6 points) If $A =$ **(CHOMP)**, then $\operatorname{rank}(A) =$ ______ and $\operatorname{nullity}(A) =$ ______.

12. (6 points)

(a) (**Fill-In.**) If $L\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by **(CHOMP)**, then $\dim(\ker L) =$ ______ and $\dim(\text{image } L) =$ ______.

(b) (**W**) Is $L$ one-to-one? Why or why not?

(c) (**W**) Is $L$ onto? Why or why not?

13. (5 points) (**W**) Find the matrix of the linear transformation $L\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined by **(CHOMP)**.

14. (4 points) (**Fill-In.**) Let $L$: **(CHOMP)** be a linear transformation. The **size** of the matrix of $L$ is

$$\_\_\_\_ \times \_\_\_\_$$

15. (6 points) (**W**) Let $W$ be the subspace of $\mathbb{R}^4$ with basis **(CHOMP)**. Use the Gram-Schmidt process to find

    (a) an orthogonal basis for $W$

    (b) an orthonormal basis for $W$.

16. (6 points) (**W**) Let $L\colon P_2 \to P_1$ be defined by **(CHOMP)**. Prove that $L$ is a linear transformation.

17. (5 points) **Multiple Choice.** *Circle the letter of the best answer.* The axes of the ellipse **(CHOMP)** lie along the vectors

    **(CHOMP)**

18. (6 points) **Fill-In.** For the matrix $A =$ **(CHOMP)**, fill in the eigenvalues and corresponding eigenvector(s) of $A$ in the table below.

    *Please put a distinct eigenvalue in each row. If there are fewer than three distinct eigenvalues, leave the last row(s) blank.*

| Eigenvalue | Corresponding eigenvector(s) |
|---|---|
| | |
| | |
| | |

19. (5 points) **(W)** Prove that if $A$ and $B$ are $n \times n$ invertible matrices, then **(CHOMP)**.

20. (5 points) **Fill-In.** The real quadratic form associated with the hyperbola **(CHOMP)** is $\mathbf{x}^T A\mathbf{x}$ where

$$A = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}.$$