Matrix Multiplication Worksheet

Recall that the *dot product* of two n-vectors

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

is $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + \ldots + a_nb_n = \sum_{i=1}^{n} a_ib_i$.

To multiply two matrices A and B, we take each *row* of A and dot it with each *column* of B. Specifically, the (i,j)-entry of AB will be the ith row of A dotted with the jth column of B. For example, let

$$A = \begin{bmatrix} 6 & -1 & 4 \\ 0 & 1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 \\ 2 \\ -9 \end{bmatrix}$$

Then the (1,1)-entry of AB is $6 \cdot 5 + -1 \cdot 2 + 4 \cdot -9 = -8$. We have

$$AB = \begin{bmatrix} -8 \\ -25 \end{bmatrix}$$

(check to make sure you believe this).

Now try the following to check your understanding:

1. Given

$$C = \begin{bmatrix} 3 & 1 & 2 & 8 \\ 4 & 0 & -5 & 1 \\ 2 & 5 & 8 & -1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -5 & 1 \\ 1 & 0 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$$

find CD.

$$CD = \begin{bmatrix} 10 & 15 \\ -17 & -5 \\ -8 & 17 \end{bmatrix}.$$

2. Suppose A is a 10×4 matrix and B is a 4×7 matrix. What will be the size of AB? Why?

AB will be 10×7. The ij-entry of AB is the ith row of A times the jth column of B. Since A has 10 rows and B has 7 columns, AB will have 10 rows and 7 columns. □

over for more fun!

3. What must be true of the sizes of A and B in order to multiply them? (You might want to try some different-sized examples to explore this question.) Write your answer with as clear a justification as you can.

The number of columns of A must equal the number of rows of B. As stated in #2, each entry of AB, if it exists, is obtained by multiplying a row of A times a column of B. Suppose $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$. Then the ith row of A looks like

$$\begin{bmatrix} a_{i1} & \cdots & a_{in} \end{bmatrix}$$

and the jth column of B looks like

$$\begin{bmatrix} b_{1j} \\ \vdots \\ b_{pj} \end{bmatrix}.$$

In order to multiply these vectors, we must have $n = p$. □

4. Is AB always the same as BA? If so, how would you prove it? If not, give an example of matrices A and B such that $AB \neq BA$.

No, they are not the same. For one thing, the sizes may not be right. For instance, if A is a 2×5 matrix and B is a 5×4 matrix, then AB is a well-defined 2×4 matrix, but BA is not defined since the number of columns of B is not equal to the number of rows of A.

But even if the sizes match, there are many pairs of matrices that do not commute. For example, let

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 1 \\ 3 & -5 \end{bmatrix}.$$

Then

$$AB = \begin{bmatrix} 2 & 2 \\ 11 & -21 \end{bmatrix},$$

but

$$BA = \begin{bmatrix} 1 & 4 \\ 11 & -20 \end{bmatrix};$$

thus we see that $AB \neq BA$. □