Matrix Multiplication Worksheet

Recall that the *dot product* of two n-vectors

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

is $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n = \sum_{i=1}^{n} a_i b_i$.

To multiply two matrices A and B, we take each *row* of A and dot it with each *column* of B. Specifically, the (i, j)-entry of AB will be the ith row of A dotted with the jth column of B. For example, let

$$A = \begin{bmatrix} 6 & -1 & 4 \\ 0 & 1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 \\ 2 \\ -9 \end{bmatrix}$$

Then the (1,1)-entry of AB is $6 \cdot 5 + -1 \cdot 2 + 4 \cdot -9 = -8$. We have

$$AB = \begin{bmatrix} -8 \\ -25 \end{bmatrix}$$

(check to make sure you believe this).

Now try the following to check your understanding:

1. Given

$$C = \begin{bmatrix} 3 & 1 & 2 & 8 \\ 4 & 0 & -5 & 1 \\ 2 & 5 & 8 & -1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -5 & 1 \\ 1 & 0 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$$

find CD.

2. Suppose A is a 10×4 matrix and B is a 4×7 matrix. What will be the size of AB? Why?

over for more fun!

3. What must be true of the sizes of A and B in order to multiply them? (You might want to try some different-sized examples to explore this question.) Write your answer with as clear a justification as you can.

4. Is AB always the same as BA? If so, how would you prove it? If not, give an example of matrices A and B such that $AB \neq BA$.