1. Find the quotient and remainder of $-200$ divided by 19.

2. Convert $(12345)_6$ from base 6 to decimal notation.

3. Convert 2007 from decimal to base 7 notation.

4. How many elements of the set $S = \{1, 2, \ldots, 2012, 2013\}$ are divisible by 3 but not by 5 nor 7?

5. Prove that $(7a + 2, 3a + 1) = 1$ for all $a \in \mathbb{Z}$.

6. Let $a \in \mathbb{Z}$. What is $(2a, 3a - 1)$?

7. Find all $n \in \mathbb{N}$ such that $n^3 - 1$ is a prime.

8. Let $a, b, c \in \mathbb{Z}$, not all zero. Recall that the greatest common divisor of $a$, $b$ and $c$ (notation : $(a, b, c)$) is the largest integer that divides $a$ and $b$ and $c$.

   Prove that $(a, b, c) = (a, (b, c))$ by showing that \{a, b, c\} and \{a, (b, c)\} have the same common divisors.

9. This exercise is about writing integers in a negative base.

   (a) Let $b \in \mathbb{Z}$ with $b < -1$. Prove that every non-zero integer $n$ can be written as

   $$n = a_kb^k + a_{k-1}b^{k-1} + \cdots + a_2b^2 + a_1b + a_0$$

   where $a_i \in \{0, 1, \ldots, |b| - 1\}$ for $0 \leq i \leq k$ and $a_k \neq 0$. You do not need to prove uniqueness.

   (b) Illustrate by writing 2013 in base $b = -2$.

10. Let $n \in \mathbb{N}$. Prove that $n! + 1$ has a prime divisor bigger than $n$.

    Deduce that there are infinitely many primes.