Math 77 Sample Exam 3 Questions, Spring 1998, Dr. Cleary

Q 1 Use symmetry to decide which of the following triple integrals are zero for
a) the sphere $x^2 + y^2 + z^2 = 1$? b) the hemisphere $x^2 + y^2 + z^2 = 1, x > 0$?
c) the half- cylinder $x^2 + y^2 = 1, y > 0$ for z between -3 and +3?
d) the cylinder $x^2 + y^2 = 1$ for z between -3 and +5?

$$\iiint_V x^2 y\, dV, \quad \iiint_V x^2 y^2\, dV, \quad \iiint_V zx^2 y\, dV, \quad \iiint_V (xyz)^2\, dV,$$

$$\iiint_V xy^2 z^2\, dV, \quad \iiint_V x^2 zy^2\, dV, \quad \iiint_V \cos(x)(2z+y)\, dV,$$

$$\iiint_V \sin(x)(2z+y)\, dV, \quad \iiint_V \sin(x)(2z+y)^2\, dV, \quad \iiint_V x^2 zy^2\, dV,$$

Q 2 Set up an integral to find the surface area of the surface given by $z = 3x^2 + 2y^2$ for $0 < x < 1$, $0 < y < 1$.

Q 3 A curved block of metal has non-uniform density. If the metal is bounded by the surface $z = 16 - x^2 - y^2$ and the xy plane, and the density at a point (x,y,z) is given (3x+3y+4z) grams/cm^2, set up an integral to find the mass of the piece.

Q 4 Let R be the surface: $\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} = 1$. Do the set up to find the surface area of R; you do not need to actually integrate.

Q 5 Is the following vector field the gradient of some scalar function on the region on which it is defined? If so, find such a scalar function.
$\vec{F} = (6x^2y^2 + \ln(x))\hat{\mathbf{i}} + (4x^3y + y\cos(y))\hat{\mathbf{j}}$.

Q 6 For the curve C given by $x = \tan^2\theta, y = 1 - \sec^2\theta, \theta \in [0, \frac{\pi}{4}]$. Compute $\int_C \vec{F} \cdot \vec{\mathbf{T}}\, ds$ for $\vec{F} = \sin x\hat{\mathbf{i}} - \cos y\hat{\mathbf{j}}$ (Hint: you may want to reparameterize the curve.)

Q 7 Tom Sawyer's aunt asked him to whitewash both sides of a fence. Tom can get a nickel for each 25 square feet of the fence he lets someone whitewash for him. If the fence's base is given by $x = 30\cos^3 t, y = 30\sin^3 t$, for t in $[0, \frac{\pi}{2}]$ and its height is given by $1 + \frac{y}{3}$, how much can he expect to make from his friends?

Q 8 If $f = x^2y + z$ compute: $\int_C \nabla f \cdot \vec{\mathbf{T}}\, ds$ for C the path given by $x = -3 + 3\cos t$, $y = 4 + 2\sin t$ for t going from 0 to 6π. Hint: the answer is zero. Why?

Q 9 Which of the following make sense? $\vec{F}$ is a vector field and f is a scalar function. For those that do make sense, what kind of function are they? Which of them are always 0?

$\nabla \times \vec{\mathbf{F}}$	$\nabla(\nabla \cdot \vec{\mathbf{F}})$	$\nabla \times \nabla(\nabla^2 f)$
$\nabla \times \nabla f$	$\nabla \times \nabla \cdot f$	$\nabla \times \nabla^2 \vec{\mathbf{F}}$
$\nabla \times \nabla \times \vec{\mathbf{F}}$	$\nabla \times \nabla^2 f$	$\nabla \cdot \nabla \vec{\mathbf{F}}$
∇f^2	$\nabla \cdot \vec{\mathbf{F}}^2$	$f\nabla \cdot f\vec{\mathbf{F}}$
$\nabla \lvert\vec{\mathbf{F}}\rvert$	$\nabla(\lvert\vec{\mathbf{F}} \times \vec{\mathbf{F}}\rvert)$	$\nabla \times ((\lvert\vec{\mathbf{F}} \times \vec{\mathbf{F}}\rvert)\hat{\mathbf{i}})$
$\nabla \cdot ((\lvert\vec{\mathbf{F}} \times \vec{\mathbf{F}}\rvert) f\vec{\mathbf{F}})$	$\nabla \times ((\lvert\vec{\mathbf{F}} \times \vec{\mathbf{F}}\rvert) \times f\vec{\mathbf{F}})$	$\nabla \times ((\lvert\vec{\mathbf{F}} \times \vec{\mathbf{F}}\rvert)\lvert\vec{\mathbf{F}}\rvert)$

Q 10 Consider the curve $\vec{R}(t) = \cos^3 t\,\hat{\mathbf{i}} + \sin^3 t\,\hat{\mathbf{j}}$ for t in $[0, 2\pi]$. Find $\int_C \vec{\mathbf{F}} \cdot \vec{\mathbf{T}}\, ds$ for $\vec{\mathbf{F}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

Q 11 Suppose $\vec{\mathbf{F}} = \nabla f$ and $\vec{\mathbf{F}} = \nabla g$. What can be said about the relationship between f and g? Prove what you claim.

Q 12 Bob the Baker decides to bake a cake for his math professor. He has a circular cake pan which is 8 inches across, centered at the origin. He makes a tasty chocolate cake and he decides to frost the top with orange frosting and the sides with lemon frosting. If the height in inches of the cake at a point (x, y) is given by $4 + x^2 + y$, what is a definite integral equal to the amount of lemon frosting needed to frost the cake, in square inches?

True/False/Meaningless:

Q 13 $\nabla \times (\vec{F} \cdot \vec{G}) = 0$

Q 14 $\nabla^2(f + g) = \nabla^2 f + \nabla^2 g$

Q 15 $\nabla \times (\nabla \cdot \nabla f) = 0$

Q 16 $\nabla \times (\nabla(\vec{F} \cdot \vec{G})) = 0$

Q 17 $\nabla^2(3x + xy) = \nabla \cdot (\tan(y)\hat{\mathbf{i}} + \sin(x)\cos(x)\hat{\mathbf{j}})$

Q 18 $\nabla \times \nabla^2(\nabla(\vec{F} \cdot \hat{\mathbf{i}})) = 0$

Q 19 $\frac{\partial}{\partial x}(\vec{F} \cdot \hat{\mathbf{i}}) + \frac{\partial}{\partial y}(\vec{F} \cdot \hat{\mathbf{j}}) + \frac{\partial}{\partial z}(\vec{F} \cdot \hat{\mathbf{k}}) = \nabla \cdot \vec{F}$

Q 20 Lori the cyclist is riding her bike straight from her home at (2,0) to her classroom at (0,6) on a windy day. The vector field describing the flow of the wind at the position (x,y) is given by $\vec{F} = 2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$
How much work does she have to do against the wind during her cycle ride? In particular, does the wind in general push her along or is she mostly riding against the wind?

Summary of expectations about recent material:
Make sure you know how to do double integrals to find surface area. Make sure you know how to do triple integrals using rectangular, cylindrical and spherical coordinates. Understand general coordinate systems and the change of variables using the Jacobian. Understand vector fields: how to plot them, all vector differential operators and how to interpret work integrals. Know how to set up line integrals in the plane and space, with respect to ds, dx, dy and dz. Know how to compute work integrals and understand the connections between path-independence and conservative vector fields. Be able to find the potential for a conservative vector field $\vec{F}$