Math 111, Fall 2014 - Extra Credit

Due Thursday, December 11, 2014, by 5:00 p.m.

You must show all of your work and explain all of your answers to receive full credit.

- The function f : Z × Z → Z × Z defined by f(m, n) = (5m + 4n, 4m + 3n) is bijective. Find its inverse. You do <u>not</u> need to prove that the function is bijective.
 Solution:
- 2. Let $A = \{x \in \mathbb{R} : x \ge 1\}$ and $B = \{x \in \mathbb{R} : x > 0\}$. For each function below, determine $f(A), f^{-1}(A), f^{-1}(B), f^{-1}(\{1\})$.
 - (a) $f : \mathbb{R} \to B$ defined by $f(x) = e^{x^3 + 1}$
 - (b) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$

Solution:

- 3. Given a function $f: C \to Z$ and sets $A, B \subseteq C$ and $X, Y \subseteq Z$.
 - (a) Prove or dispove: $f(A \cap B) = f(A) \cap f(B)$.
 - (b) Prove or disprove: $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$.

Solution: