# Math 111, Fall 2014 - Extra Credit <br> Due Thursday, December 11, 2014, by 5:00 p.m. 

## You must show all of your work and explain all of your answers to receive full credit.

1. The function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ defined by $f(m, n)=(5 m+4 n, 4 m+3 n)$ is bijective. Find its inverse.
You do not need to prove that the function is bijective.

## Solution:

2. Let $A=\{x \in \mathbb{R}: x \geq 1\}$ and $B=\{x \in \mathbb{R}: x>0\}$. For each function below, determine $f(A), f^{-1}(A), f^{-1}(B), f^{-1}(\{1\})$.
(a) $f: \mathbb{R} \rightarrow B$ defined by $f(x)=e^{x^{3}+1}$
(b) $f: \mathbb{R} \rightarrow \mathbb{R}$ defind by $f(x)=x^{2}$

## Solution:

3. Given a function $f: C \rightarrow Z$ and sets $A, B \subseteq C$ and $X, Y \subseteq Z$.
(a) Prove or dispove: $f(A \cap B)=f(A) \cap f(B)$.
(b) Prove or disprove: $f^{-1}(X \cap Y)=f^{-1}(X) \cap f^{-1}(Y)$.

Solution:

