

Math 111, Fall 2014 - Homework # 1

Remember that you are required to fully explain all of your solutions.

1. Write the following sets by listing their elements within braces.

(a) $A = \{x \in \mathbb{R} : x^2 - x = 0\}$

(b) $B = \{n \in \mathbb{Z} : n^2 < 7\}$

(c) $C = \{x \in \mathbb{R} : x^2 + 1 = 0\}$

(d) $D = \{3n + 1 : n \in \mathbb{Z}\}$

Solution:

2. Write each of the following sets in the form $\{x \in S : p(x)\}$ or $\{p(x) : x \in S\}$, where $p(x)$ is a property concerning x and S is the set containing x .

(a) $A = \{1, 2, 3, 4, \dots, 9\}$

(b) $B = \{\dots, -8, -4, 0, 4, 8, \dots\}$

(c) $C = \{1, 8, 27, 64, \dots\}$

Solution:

3. Give an example of three sets A , B , and C such that $A \in B$ and $A \subseteq C$, or state why such an example cannot exist.

Solution:

4. Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$ for $A = \{0, \{1\}, \{1, 2\}, \{\emptyset\}\}$.

Solution:

5. True or False: If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$ but $\{1\} \notin A$.

Solution:

6. True or False: If a set B has one more element than a set A , then $\mathcal{P}(B)$ has at least two more elements than $\mathcal{P}(A)$.

Solution:

7. For the sets $A = \{1, \{1\}\}$ and $B = \{0, 1, 2\}$, write down all of the elements of $A \times B$. What is $|A \times B|$?

Solution:

8. For the set $A = \{1, 2\}$ and $B = \{\emptyset\}$, write down all of the elements of $A \times B$ and $\mathcal{P}(A) \times \mathcal{P}(B)$.

Solution:

9. Describe the graph of the ellipse $4x^2 + 9y^2 = 36$ as a subset of $\mathbb{R} \times \mathbb{R}$.

Note: What I'm looking for here is something like:

The ellipse $4x^2 + 9y^2 = 36$ is the set

$$\{(x, y) \in \mathbb{R} \times \mathbb{R} : \text{_____}\}$$

(Now you fill in the blank.)

Solution: