Math 111, Fall 2014 - Homework # 12

Due Monday, December 8, 2014, by 3:00 p.m.

You must show all of your work and explain all of your answers to receive full credit.

1. Give an example of a relation from $\mathbb{Z}$ to $\{0, 1\}$ that is **not** a function.

   Solution:

2. Suppose $f : \mathbb{N} \cup \{0\} \to \mathbb{Z}$ is defined as $f = \{(x, 4\sqrt{x} - 5) : x \in \mathbb{Z}\}$. State the domain, codomain, and range of f.

   Solution:

3. Consider functions from $\mathbb{Z}$ to $\mathbb{Z}$. Give an example of

   (a) a function that is injective but not surjective;

   (b) a function that is surjective but not injective; and

   (c) a function that is neither injective nor surjective.

   For each example, prove that your function satisfies the given property.

   Solution:

4. Prove that the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \left(\dfrac{x+1}{x-1}\right)^3$ is bijective.

   Solution:

5. Suppose that A, B, and C are nonempty sets and $f : A \to B$ and $g : B \to C$.

   (a) Prove or disprove. If $g \circ f$ is injective, then f is injective.

   (b) Prove or disprove. If $g \circ f$ is injective, then g is injective.

   Solution:

6. Consider the functions $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f(m, n) = m + n$ and $g : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $g(m) = (m, m)$. Find formulas for $g \circ f$ and $f \circ g$.

   Solution: