Math 111, Fall 2014 - Homework # 3

Due Thursday, September 18, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

- 1. For a real number r, define $A_r = \{r^2\}$, B_r as the closed interval [r-1, r+1], and C_r as the interval (r, ∞) . For $S = \{1, 2, 4\}$, determine
 - (a) $\bigcup_{r \in S} A_r$ and $\bigcap_{r \in S} A_r$ (b) $\bigcup_{r \in S} B_r$ and $\bigcap_{r \in S} B_r$ (c) $\bigcup_{r \in S} C_r$ and $\bigcap_{r \in S} C_r$.

Solution:

- 2. For $r \in \mathbb{R}^+$ ($\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$), let $A_r = \{x \in \mathbb{R} : |x| < r\}$. Determine $\bigcup_{r \in \mathbb{R}^+} A_r$ and $\bigcap_{r \in \mathbb{R}^+} A_r$. Solution:
- 3. Determine which of the following are statements. For statements, determine if they are true or false.
 - (i) Every even integer is a real number.
 - (ii) $\mathbb{N} \notin P(\mathbb{N})$.
 - (iii) The integer x is divisible by 5.
 - (iv) $\emptyset = \{\emptyset\}.$

Solution:

- 4. Express each statement or open sentence in one of the forms $P \wedge Q$, $P \vee Q$, or $\sim P$. Make sure to state exactly what statements P and Q stand for.
 - (i) The matrix A is not invertible.
 - (ii) x < y
 - (iii) At least one of the numbers x and y equals 0.
 - (iv) $x \in A \cap B$

Solution:

- 5. State the negation of each of the following statements without using the word "not."
 - (a) The real number r is at most 2.
 - (b) The absolute value of the number a is less than 3.
 - (c) Two sides of the triangle have the same length.
 - (d) No one expected it to rain.
 - (e) It is surprising that two students received the same exam score.

Solution:

- 6. Consider the statements P: 17 is even and Q: 19 is prime. Write each of the following statements in words and indicate whether it is true or false.
 - (a) $\sim P$
 - (b) $P \wedge Q$
 - (c) $P \lor Q$

Solution:

- 7. Without changing their meanings, convert each of the following sentences into a sentence having the form "If P, then Q."
 - (a) Whenever three sides of a triangle are equal, the angles of the triangle are equal.
 - (b) The square of every integer is positive.
 - (c) The integer n^3 is even only if n is even.

Solution:

- 8. Without changing their meanings, convert each of the following sentences into a sentence having the form "P if and only if Q."
 - (a) If a function has constant derivative, it is linear, and conversely.
 - (b) For a circle to have both a perimeter and an area of 4π , it is necessary and sufficient that its radius be 2.

Solution:

- 9. Consider the statements $P: \sqrt{2}$ is rational and $Q: \frac{22}{7}$ is rational. Write each of the following statements in words and indicate whether it is true or false.
 - (a) $P \implies Q$
 - (b) $Q \implies P$
 - (c) $P \iff Q$

Solution: