# Math 111, Fall 2014 - Homework \# 3 <br> Due Thursday, September 18, 2014, by 4:30 p.m. 

## Remember that you are required to fully explain all of your solutions.

1. For a real number $r$, define $A_{r}=\left\{r^{2}\right\}, B_{r}$ as the closed interval $[r-1, r+1]$, and $C_{r}$ as the interval $(r, \infty)$. For $S=\{1,2,4\}$, determine
(a) $\bigcup_{r \in S} A_{r}$ and $\bigcap_{r \in S} A_{r}$
(b) $\bigcup_{r \in S} B_{r}$ and $\bigcap_{r \in S} B_{r}$
(c) $\bigcup_{r \in S} C_{r}$ and $\bigcap_{r \in S} C_{r}$.

## Solution:

2. For $r \in \mathbb{R}^{+}\left(\mathbb{R}^{+}=\{x \in \mathbb{R}: x>0\}\right)$, let $A_{r}=\{x \in \mathbb{R}:|x|<r\}$. Determine $\bigcup_{r \in \mathbb{R}^{+}} A_{r}$ and $\bigcap_{r \in \mathbb{R}^{+}} A_{r}$.

## Solution:

3. Determine which of the following are statements. For statements, determine if they are true or false.
(i) Every even integer is a real number.
(ii) $\mathbb{N} \notin P(\mathbb{N})$.
(iii) The integer $x$ is divisible by 5 .
(iv) $\varnothing=\{\varnothing\}$.

## Solution:

4. Express each statement or open sentence in one of the forms $P \wedge Q, P \vee Q$, or $\sim P$. Make sure to state exactly what statements $P$ and $Q$ stand for.
(i) The matrix $A$ is not invertible.
(ii) $x<y$
(iii) At least one of the numbers $x$ and $y$ equals 0 .
(iv) $x \in A \cap B$
5. State the negation of each of the following statements without using the word "not."
(a) The real number $r$ is at most 2 .
(b) The absolute value of the number $a$ is less than 3 .
(c) Two sides of the triangle have the same length.
(d) No one expected it to rain.
(e) It is surprising that two students received the same exam score.

## Solution:

6. Consider the statements $P: 17$ is even and $Q: 19$ is prime. Write each of the following statements in words and indicate whether it is true or false.
(a) $\sim P$
(b) $P \wedge Q$
(c) $P \vee Q$

## Solution:

7. Without changing their meanings, convert each of the following sentences into a sentence having the form "If $P$, then $Q$."
(a) Whenever three sides of a triangle are equal, the angles of the triangle are equal.
(b) The square of every integer is positive.
(c) The integer $n^{3}$ is even only if $n$ is even.

## Solution:

8. Without changing their meanings, convert each of the following sentences into a sentence having the form " $P$ if and only if $Q$."
(a) If a function has constant derivative, it is linear, and conversely.
(b) For a circle to have both a perimeter and an area of $4 \pi$, it is necessary and sufficient that its radius be 2 .

## Solution:

9. Consider the statements $P: \sqrt{2}$ is rational and $Q: \frac{22}{7}$ is rational. Write each of the following statements in words and indicate whether it is true or false.
(a) $P \Longrightarrow Q$
(b) $Q \Longrightarrow P$
(c) $P \Longleftrightarrow Q$

## Solution:

