Math 111, Fall 2014 - Homework # 4

Due Thursday, September 25, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

1. Write a truth table for the following statements.

   (a) $(Q \vee R) \iff (R \wedge Q)$

   (b) $\sim (P \wedge Q) \wedge (\sim P)$

   (c) $P \vee (Q \wedge \sim R)$

   (d) $(P \implies Q) \implies (\sim P)$

   Solution:

2. Suppose the statement $((P \wedge Q) \vee R) \implies (R \vee S)$ is false. Without using a truth table, determine the truth values of P, Q, R, and S.

   Solution:

3. Use truth tables to show that the following statements are equivalent.

   (a) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

   (b) $\sim P \iff Q = (P \implies \sim Q) \wedge (\sim Q \implies P)$

   Solution:

4. Determine whether or not the following pairs of statements are logically equivalent.

   (a) $\sim (P \implies Q)$ and $P \wedge \sim Q$

   (b) $(\sim Q) \implies (P \wedge \sim P)$ and Q

   (c) $(P \wedge Q) \iff P$ and $P \implies Q$

   (d) $\sim (P \vee Q)$ and $(\sim P) \vee (\sim Q)$

   Solution:

5. Write the following as English sentences. State whether they are true or false.

   (a) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$

   (b) $\exists n \in \mathbb{N}, \forall X \in \mathcal{P}(\mathbb{N}), |X| < n$

   (c) $\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n$

   (d) $\forall X \in \mathcal{P}W(\mathbb{N}), X \subseteq \mathbb{R}$

   Solution:

6. Translate each of the following sentences into symbolic logic.

   (a) The number x is positive and the number y is positive.

   (b) For every positive number ϵ there is a positive number M for which $|f(x)-b| < \epsilon$, whenever $x > M$.

   (c) There exist integers a and b such that both $ab < 0$ and $a + b > 0$.

   (d) For all real numbers x and y, $x \neq y$ implies that $x^2 + y^2 > 0$.

   (e) If $\sin x < 0$, then it is not the case that $0 \leq x \leq \pi$.

   Solution:

7. Let $P(x)$ and $Q(x)$ be open sentences where the domain of the variable x is T. Which of the following implies that $P(x) \implies Q(x)$ is true for all $x \in T$?

   (a) $P(x) \wedge Q(x)$ is false for all $x \in T$.

   (b) $Q(x)$ is true for all $x \in T$.

   (c) $P(x)$ is false for all $x \in T$.

   (d) $P(x) \wedge (\sim Q(x))$ is true for some $x \in T$.

   (e) $(\sim P(x)) \wedge (\sim Q(x))$ is false for all $x \in T$.

   Solution: