Math 111, Fall 2014 - Homework # 5

Due Thursday, October 2, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

1. Write the negation of the following sentences.

   (a) Either $x = 0$ or $y = 0$.

   (b) The integers a and b are both nonnegative.

   (c) If f is a polynomial and its degree is greater than 1, than f' is not constant.

   (d) There exists a rational number r such that $r^2 = 2$.

   (e) If xy is even, then either x or y is even.

   (f) For every rational number r, the number $\dfrac{1}{r}$ is rational.

   Solution:

2. Let $n \in \mathbb{Z}$. For which implication is its negation the following?
   The integer $3n + 4$ is odd and $5n - 6$ is even.

   Solution:

3. Find the number of possible user passwords with 7 characters that consist of digits or letters of the alphabet, without repetition.

   Solution:

4. How many 7-digit numbers can be made from the digits 1, 2, 3, 4, 5, 6, 7 if there is no repetition and the odd digits must appear in an unbroken sequence. (So, 1357246 and 2753146 satisfy this condition, but 7654231 does not.)

   Solution:

5. Suppose A is a set such that $|A| = 100$.

   (a) How many subsets of A have 5 elements?

   (b) How many subsets have 10 elements?

   (c) How many have 99 elements?

   Solution:

6. Determine
$$|\{X \in P(\{0,1,2,3,4,5,6,7,8,9\}) : |X| = 4\}|.$$

Solution:

7. This problem concerns lists made from the symbols $A, B, C, D, E, F, G, H, I$.

   (a) How many length-5 lists can be made if repetition is not allowed and the list must begin with a vowel?

   (b) How many length-5 lists can be made if repetition is not allowed and the list must contain exactly one A?

   (c) How many length-5 lists can be made if repetition is not allowed and the list is in alphabetical order? (For example, $BDEFI$ or $ABCGH$ is allowed, but $BACGH$ is not allowed.)

   (d) How many length-5 lists can be made if repetition is not allowed and the list is **not** in alphabetical order?

Solution:

8. Suppose a set B has the property that $|\{X : X \in P(B), |X| = 6\}| = 28$. What is $|B|$?

Solution: