

Math 111, Fall 2014 - Homework # 8

Due Thursday, October 30, 2014, by 4:30 p.m.

Remember that you are required to fully explain all of your solutions.

Prove each of the following statements.

1. Suppose $x \in \mathbb{Z}$. Then x is odd if and only if $3x + 5$ is even.

Solution:

2. Suppose $x, y \in \mathbb{R}$. Then $x^3 + x^2y = y^2 + xy$ if and only if $y = x^2$ or $y = -x$.

Solution:

3. Suppose $a, b \in \mathbb{Z}$. Then $a \equiv b \pmod{10}$ if and only if $a \equiv b \pmod{2}$ and $a \equiv b \pmod{5}$.

Solution:

4. There exists a positive real number x for which $x^2 < \sqrt{x}$.

Solution:

5. There is a set X such that $\mathbb{N} \in X$ and $\mathbb{N} \subseteq X$.

Solution:

6. Suppose $a, b \in \mathbb{N}$. Then $a = \text{lcm}(a, b)$ if and only if $b \mid a$.

Solution:

7. For every real number x , there exist integers a and b such that $a \leq x \leq b$ and $b - a = 1$.

Solution: