

Exact Equations – Section 1.6

Math 81, Applied Analysis
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1 Description and Solution Method

Often, the general solution of a differential equation is defined implicitly by an equation of the form

$$F(x, y(x)) = c, \quad c = \text{constant} \quad (1)$$

Given (1), we can reconstruct the differential equation by differentiating with respect to x :

In other words, the differential equation is of the form:

It is convenient to write (2) as

(called the differential form)

(2), or equivalently (3), defines an **exact differential equation** with solution $F(x, y) = c$ **if** $M(x, y) = \frac{\partial F}{\partial x}$ and $N(x, y) = \frac{\partial F}{\partial y}$.

Question: When is (2) (or (3)) exact?

Answer:

By assumption of the continuity of the first partial derivatives,

If (2) (or, equivalently (3)) is exact, we can solve for $F(x, y)$:

1. Integrate $M(x, y)$ with respect to x (viewing y as constant) to obtain:

$$F(x, y) = \int M(x, y)dx + g(y).$$

Then, using the fact that $N(x, y) = \frac{\partial F}{\partial y}$, solve for $g(y)$.

2. Integrate $N(x, y)$ with respect to y (viewing x as constant) to obtain:

$$F(x, y) = \int N(x, y)dy + h(x).$$

Then, using the fact that $M(x, y) = \frac{\partial F}{\partial x}$, solve for $h(x)$.

2 Examples

1) Find a general solution of

$$(2x^2 - y^2)dx + (2y^2 - 2xy)dy = 0$$

2) Solve the initial value problem

$$y^3 + 3xy^2 \frac{dy}{dx} = 0, \quad y(1) = -2$$