Translation and Partial Fractions – Section 10.3

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We have already discussed the the main idea of what you will need to know in terms of partial fractions. Our discussion, therefore, will focus on translation on the s-axis.

Theorem 1 (Translation on the s-axis)**.** *If $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > c$, then $\mathcal{L}\{e^{at} f(t)\}$ exists for $s > a + c$, and*

Proof.

□

Examples:

(1) Find $\mathcal{L}\{e^{-3t}\cos t\}$.

(2) Find

$$\mathcal{L}^{-1}\left\{\frac{6}{(s-4)^3}\right\}$$

(3) Find

$$\mathcal{L}^{-1}\left\{\frac{s-3}{s^2-6s+13}\right\}$$

(4) Find

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+4s+6}\right\}$$

(5) Find

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2-2s+1}\right\}$$

We will now apply this to solving an initial value problem.

Example: Solve the initial value problem

$$y'' + 4y' + 5y = 0,\ y(0) = 2,\ y'(0) = 0$$