Linear First Order Differential Equations and Applications – Section 1.5

Math 81, Applied Analysis
Instructor: Dr. Doreen De Leon

1 Definition and Examples

Definition A first-order differential equation is **linear** if it can be written in the form

Notice that the equation must be linear in y and y' to be a linear differential equation, but $p(x)$ and $q(x)$ can be any function.

Definition:

- If $q(x) \equiv 0$, then (1) is **homogeneous**.
- If $q(x)$ is not identically 0, then (1) is **nonhomogeneous**

Examples:

Other examples of nonlinear differential equations are

Solution to (1):

Examples

1) Find a general solution of

$$(x^2 + 1)y' + 3xy = 6x.$$

2) Solve the IVP

$$y' - y = \frac{8}{11}e^{-\frac{x}{3}},\ y(0) = -1.$$

2 Application – Mixture Problem

Example: A tank initially contains 1,000 gal of water in which is dissolved 20 lb of salt. A valve is opened and water containing 0.2 lb of salt per gallon flows into the tank at a rate of 5 gal/min. Assume that the mixture is kept uniform and drains from the tank at a rate of 5 gal/min. Howe long will it be before the tank contains twice the salt?

Mathematical Model:

rate of change in amount of salt in the tank = rate salt enters - rate salt leaves
(i.e., if $y(t)$ = amount of salt in the tank at time t, y' = rate in - rate out)

You must keep in mind that in order to determine the "rate in" and "rate out," you must multiply the concentration of the salt that enters/leaves the tank by the volume of fluid in the tank. Although the volume of fluid in the tank is constant in this example (since the rate at which fluid enters the tank is the same as the rate at which fluid drains from the tank), this is not always true.

Solution: