

Separable Equations and Applications – Section 1.4

Math 81, Applied Analysis
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1 Definition and Examples

Definition: The first order differential equation

$$\frac{dy}{dx} = H(x, y)$$

is separable if it can be written in the form

Examples:

- 1) Find a general solution of $y' = 4y$.

2) Find a general solution of $y' = \frac{3y}{x}$.

3) Find a general solution of $\frac{dy}{dx} = \frac{y}{1+x^2}$

1.0.1 Initial Value Problems

Example: Solve the initial value problem:

$$y' = \frac{e^{-y}}{1+x}, \quad y(0) = 1.$$

2 Applications of Separable Equations – Modeling

- **Radiocarbon Dating**

Carbon-14 is a radioactive isotope of carbon produced in the upper atmosphere by radiation from the sun. Plants absorb carbon dioxide from the air, and living organisms eat plants. The ratio of normal carbon (Carbon-12) to Carbon-14 in the air and living things is nearly constant. When a living creature dies, the Carbon-14 begins to decrease due to radioactive decay. By comparing the amounts of Carbon-12 and Carbon-14, the amount of Carbon-14 that has decayed can be determined.

Mathematical Model

Let $y(t)$ = amount of Carbon-14 present at time t after death.

The model of radioactive decay:

We need additional information to find k . For example, if we have the half-life of Carbon-14 (which is standard information for a radioactive isotope), we can determine k .

Please note that before determining the value of k we must first (1) set up and solve the differential equation, and (2) determine the constant c using the initial information (i.e., the amount of Carbon-14 present in the body initially).

Sample Case

The half-life of Carbon-14 is approximately 5,730 years. Suppose remains have been found in which it is estimated that 30% of the original Carbon-14 is present. Estimate the age of the remains.

- See the textbook for cooling/heating and Torricelli's law problems.