Solutions to Suggested Problems - Sec 3.2-3.6
Math 81
Instructor: Dr. Doreen De Leon

- Section 3.4: 3, 8, 27
- Section 3.2: 11, 15, 17, 23
- Section 3.3: 21, 25, 27
- Section 3.5: 12, 22
- Section 3.6: 5, 19

- Section 3.4

3. $A=\begin{pmatrix}5 & 0\\ 0 & 7\\ 3 & -1\end{pmatrix}$, $B=\begin{pmatrix}-4 & 5\\ 3 & 2\\ 7 & 4\end{pmatrix}$, $c=-2$, $d=4$

$$cA+dB=\begin{pmatrix}-2(5) & -2(0)\\ -2(0) & -2(7)\\ -2(3) & -2(-1)\end{pmatrix}+\begin{pmatrix}4(-4) & 4(5)\\ 4(3) & 4(2)\\ 4(7) & 4(4)\end{pmatrix}=\begin{pmatrix}-10+16 & 0+20\\ 0+12 & -14+8\\ -6+28 & 2+16\end{pmatrix}=\begin{pmatrix}-26 & 20\\ 12 & -6\\ 22 & 18\end{pmatrix}$$

8. $A=\begin{pmatrix}1 & 0 & 3\\ 2 & -5 & 4\end{pmatrix}$, $B=\begin{pmatrix}3 & 0\\ -1 & 4\\ 6 & 5\end{pmatrix}$

$$AB=\begin{pmatrix}1 & 0 & 3\\ 2 & -5 & 4\end{pmatrix}\begin{pmatrix}3 & 0\\ -1 & 4\\ 6 & 5\end{pmatrix}=\begin{pmatrix}1\cdot 3+0(-1)+3(6) & 1(0)+0(4)+3(5)\\ 2\cdot 3+(-5)(-1)+4(6) & 2(0)+(-5)(4)+4(5)\end{pmatrix}$$

$$=\begin{pmatrix}21 & 15\\ 35 & 0\end{pmatrix}$$

$$BA=\begin{pmatrix}3 & 0\\ -1 & 4\\ 6 & 5\end{pmatrix}\begin{pmatrix}1 & 0 & 3\\ 2 & -5 & 4\end{pmatrix}=\begin{pmatrix}3\cdot 1+0\cdot 2 & 3\cdot 0+0(-5) & 3(3)+0(4)\\ -1(1)+4(2) & -1(0)+4(-5) & -1(3)+4(4)\\ 6(1)+5(1) & 6(0)+5(-5) & 6(3)+5(4)\end{pmatrix}$$

$$=\begin{pmatrix}3 & 0 & 9\\ 7 & -20 & 13\\ 11 & -25 & 38\end{pmatrix}$$

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27. Let $A = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & 0 \cdots & 0 \\ \vdots & & \ddots & \vdots \\ & & & 0 \\ 0 & \cdots & 0 & a_{nn} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ 0 & b_{22} & 0 \cdots & 0 \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & b_{nn} \end{pmatrix}$

$AB = \begin{pmatrix} a_{11}b_{11} & 0 & & 0 \\ 0 & a_{22}b_{22} & 0 \cdots & 0 \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & a_{nn}b_{nn} \end{pmatrix}$ = a diagonal matrix

Rule: Multiply corresponding entries on the diagonal

Since $ab = ba$ for scalars a and b, $a_{ii}b_{ii} = b_{ii}a_{ii}$
$\Rightarrow AB = BA$

Section 3.2

11. $2x_1 + 8x_2 + 3x_3 = 2$
$x_1 + 3x_2 + 2x_3 = 5$
$2x_1 + 7x_2 + 4x_3 = 8$

$$\left(\begin{array}{ccc|c} 2 & 8 & 3 & 2 \\ 1 & 3 & 2 & 5 \\ 2 & 7 & 4 & 8 \end{array}\right) \xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 2 & 8 & 3 & 2 \\ 2 & 7 & 4 & 8 \end{array}\right) \xrightarrow[r_3 \to r_3 - 2r_1]{r_2 \to r_2 - 2r_1} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 2 & -1 & -8 \\ 0 & 1 & 0 & -2 \end{array}\right)$$

$$\xrightarrow{r_2 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 2 & -1 & -8 \end{array}\right) \xrightarrow{r_3 \to r_3 - 2r_2} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & -4 \end{array}\right)$$

$-x_3 = -4 \Rightarrow x_3 = 4$
$x_2 = -2$
$x_1 + 3x_2 + 2x_3 = 5 \Rightarrow x_1 = 3$

$\boxed{x_1 = 3,\ x_2 = -2,\ x_3 = 4}$

15. $3x_1 + x_2 - 3x_3 = -4$
$x_1 + x_2 + x_3 = 1$
$5x_1 + 6x_2 + 8x_3 = 8$

$$\left(\begin{array}{ccc|c} 3 & 1 & -3 & -4 \\ 1 & 1 & 1 & 1 \\ 5 & 6 & 8 & 8 \end{array}\right) \xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 1 & -3 & -4 \\ 5 & 6 & 8 & 8 \end{array}\right) \xrightarrow[r_3 \to r_3 - 5r_1]{r_2 \to r_2 - 3r_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -6 & -7 \\ 0 & 1 & 3 & 3 \end{array}\right)$$

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$\xrightarrow{r_3 \leftrightarrow r_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & -2 & -6 & -7 \end{array}\right) \xrightarrow{r_3 \to r_3 + 2r_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $0 \neq -1$
$\Rightarrow$ No solution

17. $x_1 - 4x_2 - 3x_3 - 3x_4 = 4$
$2x_1 - 6x_2 - 5x_3 - 5x_4 = 5$
$3x_1 - x_2 - 4x_3 - 5x_4 = -7$

$\left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 2 & -6 & -5 & -5 & 5 \\ 3 & -1 & -4 & -5 & -7 \end{array}\right) \xrightarrow[r_3 \to r_3 - 3r_1]{r_2 \to r_2 - 2r_1} \left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 11 & 5 & 4 & -19 \end{array}\right)$

$\xrightarrow{r_3 \to r_3 - \frac{11}{2} r_2} \left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \end{array}\right) \xrightarrow{r_3 \to -2r_3} \left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 0 & 1 & 3 & 5 \end{array}\right)$

x_4 is arbitrary, so let $x_4 = t$

$x_3 + 3x_4 = 5 \Rightarrow x_3 = 5 - 3t$

$2x_2 + x_3 + x_4 = -3 \Rightarrow x_2 = \frac{1}{2}(-3 - x_3 - x_4) = \frac{1}{2}(-3 - (5 - 3t) - t))$
$= -4 + t$

$x_1 - 4x_2 - 3x_3 - 3x_4 = 4 \Rightarrow x_1 = 4 + 4x_2 + 3x_3 + 3x_4$
$= 4 + 4(-4 + t) + 3(5 - 3t) + 3t$
$= 3 - 2t$

So $\boxed{x_1 = 3 - 2t,\ x_2 = -4 + t,\ x_3 = 5 - 3t,\ x_4 = t}$

23. $3x + 2y = 1$
$6x + 4y = k$

$\left(\begin{array}{cc|c} 3 & 2 & 1 \\ 6 & 4 & k \end{array}\right) \xrightarrow{r_2 \to r_2 - 2r_1} \left(\begin{array}{cc|c} 3 & 2 & 1 \\ 0 & 0 & k \end{array}\right)$

a) Unique Solution
Not possible (since x_2 is not uniquely defined)

b) no solution
$-\infty < k < 0,\ 0 < k < \infty$ (since $0 \neq k$)

c) infinitely many solutions
$k = 0$

Section 3.3

21. See Section 3.2 #11

$$\left(\begin{array}{ccc|c} 2 & 8 & 3 & 2 \\ 1 & 3 & 2 & 5 \\ 2 & 7 & 4 & 8 \end{array}\right) \xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 2 & 8 & 3 & 2 \\ 2 & 7 & 4 & 8 \end{array}\right) \xrightarrow[r_3 \to r_3 - 2r_1]{r_2 \to r_2 - 2r_1} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 2 & -1 & -8 \\ 0 & 1 & 0 & -2 \end{array}\right)$$

$$\xrightarrow{r_2 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 2 & -1 & -8 \end{array}\right) \xrightarrow[r_3 \to r_3 - 2r_2]{r_1 \to r_1 - 3r_2} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 11 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & -4 \end{array}\right) \xrightarrow{r_3 \to -r_3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 11 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\xrightarrow{r_1 \to r_1 - 2r_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right) \Rightarrow \boxed{\begin{array}{l} x_1 = 3 \\ x_2 = -2 \\ x_3 = 4 \end{array}}$$

25. See Section 3.2 #15

$$\left(\begin{array}{ccc|c} 3 & 1 & -3 & -4 \\ 1 & 1 & 1 & 1 \\ 5 & 6 & 8 & 8 \end{array}\right) \xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 1 & -3 & -4 \\ 5 & 6 & 8 & 8 \end{array}\right) \xrightarrow[r_3 \to r_3 - 5r_1]{r_2 \to r_2 - 3r_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -6 & -7 \\ 0 & 1 & 3 & 3 \end{array}\right)$$

$$\xrightarrow{r_2 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & -2 & -6 & -7 \end{array}\right) \xrightarrow[r_3 \to r_3 + 2r_2]{r_1 \to r_1 - r_2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$0 \neq -1 \Rightarrow$ No solution

27. See Section 3.2 #17

$$\left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 2 & -6 & -5 & -5 & 5 \\ 3 & -1 & -4 & -5 & -7 \end{array}\right) \xrightarrow[r_3 \to r_3 - 3r_1]{r_2 \to r_2 - 2r_1} \left(\begin{array}{cccc|c} 1 & -4 & -3 & -3 & 4 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 11 & 5 & 4 & -19 \end{array}\right)$$

$$\xrightarrow[r_3 \to r_3 - \frac{11}{2} r_2]{r_1 \to r_1 + 2r_2} \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & -2 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \end{array}\right) \xrightarrow{r_3 \to -2r_3} \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & -2 \\ 0 & 2 & 1 & 1 & -3 \\ 0 & 0 & 1 & 3 & 5 \end{array}\right) \xrightarrow[r_2 \to r_2 - r_3]{r_1 \to r_1 + r_3}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 3 \\ 0 & 2 & 0 & -2 & -8 \\ 0 & 0 & 1 & 3 & 5 \end{array}\right) \xrightarrow{r_2 \to \frac{1}{2} r_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & 3 & 5 \end{array}\right)$$

x_4 arbitrary, so let $x_4 = t \Rightarrow x_3 + 3x_4 = 5 \Rightarrow x_3 = 5 - 3t$

$x_2 - x_4 = 1 \Rightarrow x_2 = -4+t$

$x_1 + 2x_4 = 3 \Rightarrow x_1 = 3-2t$

So $\boxed{x_1 = 3-2t,\ x_2 = -4+t,\ x_3 = 5-3t,\ x_4 = t}$

- Section 3.5

12. $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 8 & 3 \\ 3 & 10 & 6 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 2 & 8 & 3 & 0 & 1 & 0 \\ 3 & 10 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow[r_3 \to r_3 - 3r_1]{r_2 \to r_2 - 2r_1} \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{r_2 \leftrightarrow r_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 2 & -1 & -2 & 1 & 0 \end{array}\right) \xrightarrow[r_3 \to r_3 - 2r_2]{r_1 \to r_1 - 3r_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 10 & 0 & -3 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & -1 & 4 & 1 & -2 \end{array}\right)$$

$$\xrightarrow{r_3 \to -r_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 10 & 0 & -3 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & -4 & -1 & 2 \end{array}\right) \xrightarrow{r_1 \to r_1 - 2r_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 2 & -7 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & -4 & -1 & 2 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} 18 & 2 & -7 \\ -3 & 0 & 1 \\ -4 & -1 & 2 \end{pmatrix}}$$

22. $A = \begin{pmatrix} 4 & 0 & 1 & 1 \\ 3 & 1 & 3 & 1 \\ 0 & 1 & 2 & 0 \\ 3 & 2 & 4 & 1 \end{pmatrix}$

$$\left(\begin{array}{cccc|cccc} 4 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 1 & 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & 4 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{r_1 \to \frac{1}{4} r_1} \left(\begin{array}{cccc|cccc} 1 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 3 & 1 & 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & 4 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

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$$\xrightarrow{r_4 \to r_4 - r_3} \left(\begin{array}{cccc|cccc} 1 & 0 & \frac14 & \frac14 & \frac14 & 0 & 0 & 0 \\ 3 & 1 & 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 \end{array}\right) \xrightarrow{r_2 \to r_2 - 3r_1}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & \frac14 & \frac14 & \frac14 & 0 & 0 & 0 \\ 0 & 1 & \frac94 & \frac14 & -\frac34 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 \end{array}\right) \xrightarrow{r_2 \leftrightarrow r_4} \left(\begin{array}{cccc|cccc} 1 & 0 & \frac14 & \frac14 & \frac14 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & \frac94 & \frac14 & -\frac34 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[r_4 \to r_4 - r_2]{r_3 \to r_3 - r_2} \left(\begin{array}{cccc|cccc} 1 & 0 & \frac14 & \frac14 & \frac14 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & \frac54 & \frac14 & -\frac34 & 2 & 0 & -1 \end{array}\right) \xrightarrow[\substack{r_2 \to r_2 - r_3 \\ r_4 \to r_4 - \frac54 r_3}]{r_1 \to r_1 - \frac14 r_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac14 & \frac14 & -\frac14 & -\frac14 & \frac14 \\ 0 & 1 & 0 & 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & \frac14 & -\frac34 & \frac34 & -\frac54 & \frac14 \end{array}\right) \xrightarrow{r_4 \to 4r_4} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac14 & \frac14 & -\frac14 & -\frac14 & \frac14 \\ 0 & 1 & 0 & 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -3 & 3 & -5 & 1 \end{array}\right)$$

$$\xrightarrow{r_1 \to r_1 - \frac14 r_4} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -3 & 3 & -5 & 1 \end{array}\right)$$

- Section 3.6

5. Use cofactor expansion to determine $\begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 5 & 0 & 0 & 0 \end{vmatrix}$

$$\begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 5 & 0 & 0 & 0 \end{vmatrix} = (-1)^{1+3}(1)\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 5 & 0 & 0 \end{vmatrix} = (-1)^{1+1}(2)\begin{vmatrix} 0 & 3 & 0 \\ 0 & 0 & 4 \\ 5 & 0 & 0 \end{vmatrix}$$

$$= 2(3)(-1)^{1+2}\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} = 120$$

Sec 3.6 | 19. Use the method of elimination to determine

$$\begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ -2 & 3 & -2 & 3 \\ 0 & -3 & 3 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ -2 & 3 & -2 & 3 \\ 0 & -3 & 3 & 3 \end{vmatrix} \xrightarrow{r_3 \to r_3 + 2r_2} \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 3 & -2 & 9 \\ 0 & -3 & 3 & 3 \end{vmatrix} \xrightarrow[r_4 \to r_4 + 3r_2]{r_3 \to r_3 - 3r_2} \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 4 & 9 \\ 0 & 0 & -3 & 3 \end{vmatrix}$$

$$\xrightarrow{r_4 \leftrightarrow r_3} - \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 4 & 9 \end{vmatrix} \xrightarrow{r_3 \to -\frac{1}{3} r_3} -(-3) \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 4 & 9 \end{vmatrix}$$

$$\xrightarrow{r_4 \to r_4 - 4r_3} -(-3) \begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 13 \end{vmatrix} = -(-3)(1)(1)(1)(13) = \boxed{39}$$