Drug Dosage Problem Revisited

Stability and Long-term Behavior

CURM Background Material, Fall 2014

We are looking at the drug dosage problem, but for simplicity, we are assuming that the dosage of drug is 240 mg, instead of the 250 mg we assumed previously. Thus, we are analyzing the discrete dynamical system $a(n+1) = 0.70 \cdot a(n) + 240$.

First, determine the equilibrium value(s) for this system:

$ae := solve(ae = 0.70 \cdot ae + 240)$

$$800. \tag{1}$$

$drugdos1 := rsolve(\{a(n+1) = 0.70 \cdot a(n) + 240, a(0) = 0\}, a(n))$

$$800 - 800\left(\frac{7}{10}\right)^n \tag{2}$$

$d1 := unapply(drugdos1, n)$

$$n \to 800 - 800\left(\frac{7}{10}\right)^n \tag{3}$$

$drugdos2 := rsolve(\{a(n+1) = 0.70 \cdot a(n) + 240, a(0) = 240\}, a(n))$

$$-560\left(\frac{7}{10}\right)^n + 800 \tag{4}$$

$d2 := unapply(drugdos2, n)$

$$n \to -560\left(\frac{7}{10}\right)^n + 800 \tag{5}$$

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with(plots):
p1 := pointplot({seq([i, d1(i)], i = 1..20)}):
p2 := pointplot({seq([i, d2(i)], i = 1..20)}, symbol = solidcircle):
display(p1, p2)
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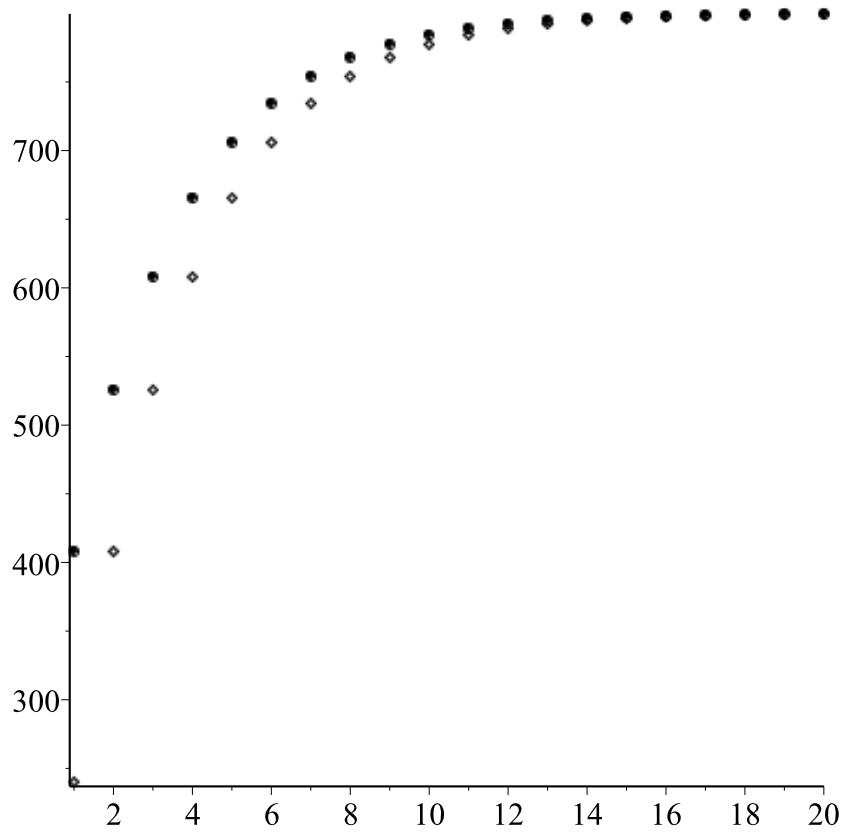

Now, check the limit to verify that the solutions converge to 800.

$\lim_{n \to \infty} d1(n)$

$$800 \quad \textbf{(6)}$$

$\lim_{n \to \infty} d2(n)$

$$800 \quad \textbf{(7)}$$

However, this is not sufficient to show that the equilibrium value of 800 is stable. To verify that the equilibrium value is stable, find the general solution and then take the limit as n goes to infinity.

$drugdos := rsolve(a(n+1) = 0.70 \cdot a(n) + 240, a(n))$

$$a(0)\left(\frac{7}{10}\right)^n + 800 - 800\left(\frac{7}{10}\right)^n \quad \textbf{(8)}$$

$d := unapply(drugdos, n)$

$$n \to a(0)\left(\frac{7}{10}\right)^n + 800 - 800\left(\frac{7}{10}\right)^n \quad \textbf{(9)}$$

$\lim_{n \to \infty} d(n)$

$$800 \quad \textbf{(10)}$$

Since the limit as n goes to infinity, regardless of initial condition, is the equilibrium value, the equilibrium value is stable.