Phys 4A Formula List

Use this to do the homework.
A copy will be available on all exams.

(Updated 1/20/2016 by F. Ringwald)

1-D kinematic equations:

$a = constant,\ t_i = 0,\ t_f = t$

$\upsilon(t) = \upsilon_i + at$

$x(t) = x_i + \upsilon_i t + \frac{1}{2}at^2$

${\upsilon_f}^2 - {\upsilon_i}^2 = 2a\,\Delta x = 2a(x_f - x_i)$

$x_f - x_i = \frac{1}{2}(\upsilon_i + \upsilon_f)t$

$\bar{\upsilon} = \frac{\upsilon_i + \upsilon_f}{2}$

$\Delta x = x_f - x_i$

$\bar{\upsilon} \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$

$\bar{\upsilon}_{speed} = \frac{d}{\Delta t}$

$\upsilon \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

$\Delta x = x_f - x_i = \int_{t_i}^{t_f} \upsilon_x(t)dt$

$\bar{a} \equiv \frac{\upsilon_f - \upsilon_i}{t_f - t_i} = \frac{\Delta \upsilon}{\Delta t}$

$a \equiv \lim_{\Delta t \to 0} \frac{\Delta \upsilon}{\Delta t} = \frac{d\upsilon}{dt} = \frac{d^2x}{dt^2}$

$\Delta \upsilon = \upsilon_f - \upsilon_i = \int_{t_i}^{t_f} a(t)dt$

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$

$|\vec{A}| = A = \sqrt{{A_x}^2 + {A_y}^2}$

$\theta = tan^{-1}\left(\frac{A_y}{A_x}\right)$

$\begin{cases} A_x = A\,cos\theta \\ A_y = A\,sin\theta \end{cases}$

$|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A}\cdot\vec{B}$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$= |\vec{A}||\vec{B}|\,cos\theta_{AB}$

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\,sin\theta_{AB}$

Solutions for x in a quadratic equation

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

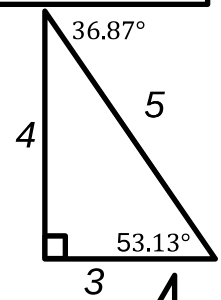
$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

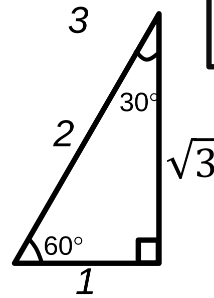
$\bar{\vec{\upsilon}} \equiv \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\Delta\vec{r}}{\Delta t}$

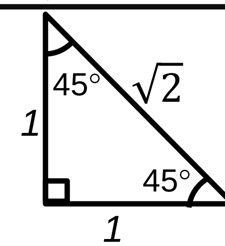
$\vec{\upsilon} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

$\bar{\vec{a}} \equiv \frac{\vec{\upsilon}_f - \vec{\upsilon}_i}{t_f - t_i} = \frac{\Delta\vec{\upsilon}}{\Delta t}$

$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}}{\Delta t} = \frac{d\vec{\upsilon}}{dt}$







$g = 9.8\frac{m}{s^2} = 980\frac{cm}{s^2}$

$1\,m = 100\,cm$
$= 1000\,mm$

$1\,kg = 1000\,g$

$T(tera) \equiv 10^{12}$
$G(giga) \equiv 10^9$
$M(mega) \equiv 10^6$
$k(kilo) \equiv 10^3$
$m(milli) \equiv 10^{-3}$
$\mu(micro) \equiv 10^{-6}$
$n(nano) \equiv 10^{-9}$
$p(pico) \equiv 10^{-12}$

Relative motion:

$\begin{cases} \vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A} \\ \vec{\upsilon}_{P,A} = \vec{\upsilon}_{P,B} + \vec{\upsilon}_{B,A} \end{cases}$

$P, A:\ P\ relative\ to\ A$

Galilean transformation:

$\vec{\upsilon}_{B,A} = \text{constant}$

$\begin{cases} \vec{r}_{P,A} = \vec{r}_{P,B} + \vec{\upsilon}_{B,A}t \\ \vec{\upsilon}_{P,A} = \vec{\upsilon}_{P,B} + \vec{\upsilon}_{B,A} \\ \vec{a}_{P,A} = \vec{a}_{P,B} \end{cases}$

For $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ = constant and $t_0 = 0$ and $t_f = t$: :

$\vec{\upsilon}(t) = \vec{\upsilon}_0 + \vec{a}t$

$\begin{cases} \upsilon_x(t) = \upsilon_{0,x} + a_xt \\ \upsilon_y(t) = \upsilon_{0,y} + a_yt \end{cases}$

$\vec{r}(t) = \vec{r}_0 + \vec{\upsilon}_0t + \frac{1}{2}\vec{a}\,t^2$

$\begin{cases} x(t) = x_0 + \upsilon_{0,x}\,t + \frac{1}{2}a_xt^2 \\ y\,(t) = y_0 + \upsilon_{0,y}t + \frac{1}{2}a_yt^2 \end{cases}$

3D spherical coordinates:

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$r = \sqrt{x^2 + y^2 + z}$

$\begin{cases} z = r\,cos\theta \\ x = r\,sin\theta\,cos\phi \\ y = r\,sin\theta\,sin\phi \end{cases}$

Uniform circular motion:

$\upsilon = \frac{2\pi r}{T},\ T = \frac{2\pi r}{\upsilon}$

$\vec{a}_c = \frac{\upsilon^2}{r}(-\hat{r}),\ \vec{F}_c = m\vec{a}_c$

Projectile motion:

$\begin{cases} \upsilon_{0,x} = \upsilon_0 cos\theta_0 \\ \upsilon_{0,y} = \upsilon_0 sin\theta_0 \end{cases}$ $\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$

symmetric trajectory:

$\begin{cases} h = \frac{{\upsilon_0}^2\,sin^2\theta_0}{2g} \\ R = \frac{{\upsilon_0}^2\,sin2\theta_0}{g} \end{cases}$

$$\vec{F}_{net} = \sum \vec{F}_n = m\vec{a}$$

$$\begin{cases} F_{net,x} = ma_x \\ F_{net,y} = ma_y \\ F_{net,z} = ma_z \end{cases}$$

Action and reaction forces:

$$\vec{\boldsymbol{F}}_{\boldsymbol{12}} = -\vec{\boldsymbol{F}}_{\boldsymbol{21}}$$

Motion of a random trajectory:

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{a}}_{\boldsymbol{r}} + \vec{\boldsymbol{a}}_{\boldsymbol{t}}$$

$$\vec{\boldsymbol{a}}_{\boldsymbol{r}} = \vec{\boldsymbol{a}}_{\boldsymbol{c}} = \frac{\boldsymbol{v^2}}{\boldsymbol{r}}\,(-\hat{\boldsymbol{r}})$$

$$\vec{\boldsymbol{a}}_{\boldsymbol{t}} = \frac{\boldsymbol{d|\vec{v}|}}{\boldsymbol{dt}}\,\hat{\boldsymbol{t}}$$

$$|\vec{\boldsymbol{a}}| = \sqrt{|\vec{\boldsymbol{a}}_{\boldsymbol{r}}|^2 + |\vec{\boldsymbol{a}}_{\boldsymbol{t}}|^2}$$

$$\vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_{\boldsymbol{r}} + \vec{\boldsymbol{F}}_{\boldsymbol{t}}$$

$$\vec{\boldsymbol{F}}_{\boldsymbol{r}} = m\vec{\boldsymbol{a}}_{\boldsymbol{r}}$$

$$\vec{\boldsymbol{F}}_{\boldsymbol{t}} = m\vec{\boldsymbol{a}}_{\boldsymbol{t}}$$

Magnitudes of frictional forces:

$$f_{s,max} = \mu_s\, N$$

$$f_k = \mu_k\, N$$

$$W = \vec{F}\cdot\Delta\vec{r} = F\,\Delta r\,cos\theta$$

$$= F_x\Delta x + F_y\Delta y + F_z\Delta z$$

or $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r}$

Hooke's law: $\vec{\boldsymbol{F}}_{\boldsymbol{sp}} = -\boldsymbol{k}\vec{\boldsymbol{x}}$

Where $x = 0$ is the equilibrium position

$$U_s = \frac{1}{2}kx^2$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$U_g = \begin{cases} mgy & ,|y| \ll R_E \\ -G\dfrac{Mm}{r} & ,r \geq R_E \end{cases}$$

$$E_{mech} = K + U_g + U_s$$

Every conservative force, such as gravitational force and spring force, has a corresponding potential energy so that $\vec{\boldsymbol{F}}_{cons} = -\vec{\nabla}U_{cons}$.

Only conservative forces, such as gravitational force and spring force, in an isolated system:

$$\textstyle\sum_n \vec{\boldsymbol{F}}_{ext,n} = 0$$

$$\Longrightarrow\ E_{mech,i} = E_{mech,f}$$

Other external forces & friction forces acting upon a system:

$$\sum_n \vec{\boldsymbol{F}}_{ext,n} \neq 0$$

$$\Longrightarrow \Delta E_{mech} = E_{mech,f} - E_{mech,i}$$

$$= W_{friction} + W_{other\ external\ forces}$$

where $W_{friction} = -f_k\, d$

$$\wp_{avg} = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}$$

$$\wp = \lim_{\Delta t\to 0}\frac{\Delta E}{\Delta t} = \frac{dE}{dt}$$

$$= \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

$$\vec{p} = m\vec{v}\,,\ \sum \vec{F}_{external} = \frac{d\vec{p}}{dt}$$

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int \vec{F}_{net,ext}\,dt$$

$$\vec{F}_{avg} = \frac{\vec{I}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

Elastic collision in 1 D (such as a head-on collision):

$$\begin{cases} v_{1,f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)v_{1,i} + \left(\dfrac{2m_2}{m_1 + m_2}\right)v_{2,i} \\ v_{2,f} = \left(\dfrac{2m_1}{m_1 + m_2}\right)v_{1,i} + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)v_{2,i} \end{cases}$$

Newton's Law of Universal Gravitation:

exerted by 1 on 2

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

$$r = |\vec{r}_2 - \vec{r}_1|$$

$$\hat{r}_{12} \equiv \frac{\vec{r}_2 - \vec{r}_1}{r}$$

$G = 6.67\times 10^{-11}$ $N\cdot m^2/kg^2$

$$U_{g,circular\ orbit} = -2K$$

$M_{Sun} = 1.9891 \times 10^{30}$ kg

$R_{Sun} = 6.955 \times 10^{8}$ m

$M_{Earth} = 5.9722 \times 10^{24}$ kg

$R_{Earth} = 6.3781 \times 10^{6}$ m

$M_{Moon} = 7.3477 \times 10^{22}$ kg

$R_{Moon} = 1.7374 \times 10^{6}$ m

Center of mass (CM) of a system:

$$M = \textstyle\sum_n m_n$$

$$\vec{r}_{CM} = \frac{\sum_n m_n\vec{r}_n}{M}$$

$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{\sum_n \vec{p}_n}{M} = \frac{\sum_n m_n\vec{v}_n}{M}$$

$$\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{\sum \vec{F}_{ext}}{M}$$

$$\vec{r}_n = \vec{r}_{CM} + \vec{r}_{n,CM}$$

$$\vec{v}_n = \vec{v}_{CM} + \vec{v}_{n,CM}$$

$$K = K_{CM} + K_{REL}$$

Rocket propulsion:

$$M\,\Delta v = v_{ex}\,\Delta m$$

Thrust:

$$M\frac{dv}{dt} = \left|v_{ex}\frac{dM}{dt}\right|$$

$$= -v_{ex}\frac{dM}{dt}$$

$$\Delta v = v_f - v_i$$

$$= v_{ex}\,ln\frac{M_i}{M_f}$$

$$\omega = \frac{d\theta}{dt}; \quad \alpha = \frac{d\omega}{dt}$$

$$s = r\theta$$

$$v = v_t = r\omega$$

$$a_t = r\alpha, \qquad a_r = r\omega^2$$

$$\vec{a} = \vec{a}_t + \vec{a}_r$$

$$|\vec{a}| = a = \sqrt{(a_t)^2 + (a_r)^2}$$

$$I = \sum_n m_n {r_n}^2$$

$$= \int r^2 dm = \begin{cases} \int r^2 \rho dV \\ \int r^2 \sigma dA \\ \int x^2 \lambda dx \end{cases}$$

$$\rho = \frac{M}{V}, \sigma = \frac{M}{A}, \lambda = \frac{M}{L}$$

Parallel-axis theorem

$$I_P = I_{CM} + MD^2$$

For α = constant and $t_i = 0$ and $t_f = t$:

$$\omega(t) = \omega_i + \alpha t$$

$$\theta(t) = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^{\ 2} - \omega_i^{\ 2} = 2\alpha\,\Delta\theta = 2\alpha(\theta_f - \theta_i)$$

$$\theta_f - \theta_i = \frac{1}{2}(\omega_i + \omega_f)t$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = |\vec{\tau}| = rF\sin\phi = rF_t = d \cdot F = I\alpha$$

$$\left(d \equiv \begin{cases} \text{moment arm} \\ \text{lever arm} \end{cases}\right)$$

$$K = K_{CM} + K_{REL} = K_{trans} + K_R$$

$$K_{CM} = K_{trans} = \frac{1}{2} M {v_{CM}}^2$$

$$K_{REL} = K_R = \frac{1}{2} I_{CM} \omega^2$$

$$\vec{\tau}_{ext,net} = \sum \vec{\tau}_{ext} = I\,\vec{\alpha}$$

$$\vec{\tau}_{ext,net} = \sum \vec{\tau}_{ext} = \frac{d\vec{L}_{tot}}{dt}$$

$$K_R = \frac{1}{2} I\omega^2 = \frac{L^2}{2\,I}$$

$$W = \tau\,\Delta\theta$$

$$\wp = \frac{dW}{dt} = \tau\,\omega$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = |\vec{L}| = r\,mv\,\sin\phi = I\omega$$

Conservation laws for an isolated system:

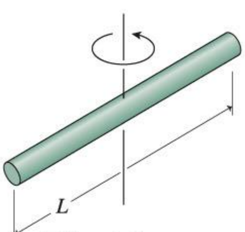
$\sum \vec{F}_{ext} = 0$ (including $f_k = 0$)

$$\Rightarrow E_{mech,\,i} = E_{mech,\,f}$$

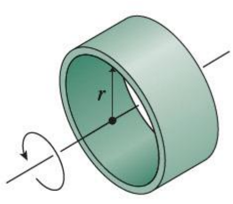
$$\sum \vec{F}_{ext} = 0 \Rightarrow (\textstyle\sum_n \vec{p}_n)_i = (\textstyle\sum_n \vec{p}_n)_f$$

$$\sum \vec{\tau}_{ext} = 0 \Rightarrow \left(\textstyle\sum_n \vec{L}_n\right)_i = \left(\textstyle\sum_n \vec{L}_n\right)_f$$

Table 10.2 Rotational Inertias

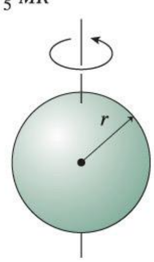


Thin rod about center
$I = \frac{1}{12} ML^2$

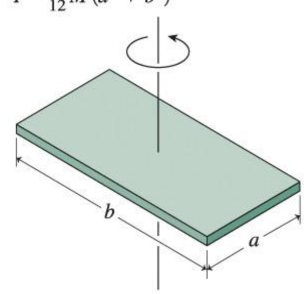


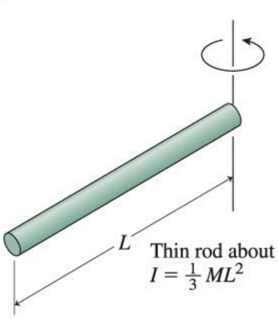
Thin ring or hollow cylinder about its axis
$I = MR^2$

Solid sphere about diameter
$I = \frac{2}{5} MR^2$

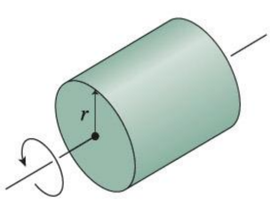


Flat plate about perpendicular axis
$I = \frac{1}{12} M(a^2 + b^2)$



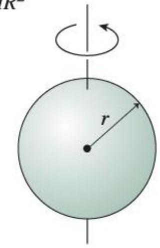


Thin rod about end
$I = \frac{1}{3} ML^2$

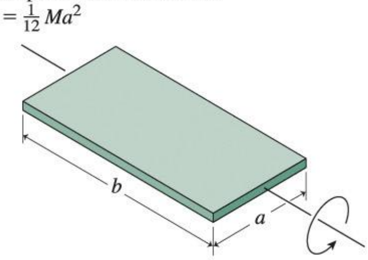


Disk or solid cylinder about its axis
$I = \frac{1}{2} MR^2$

Hollow spherical shell about diameter
$I = \frac{2}{3} MR^2$



Flat plate about central axis
$I = \frac{1}{12} Ma^2$



S.H.M. in a spring-mass system:

$$F_{net} = ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\begin{cases} x(t) = A\cos(\omega t + \phi) \\ v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \\ a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) \end{cases}$$

$$\omega = \sqrt{\frac{k}{m}}, \omega = 2\pi f, \quad f = \frac{1}{T}$$

S.H.M. of a torsional pendulum

$$\tau_{net} = -\kappa\,\theta, \ \tau_{net} = I\alpha$$

$$\Longrightarrow \frac{d^2\theta}{dt^2} \approx -\frac{\kappa}{I}\theta$$

S.H.M. of a physical pendulum

$$\tau_{net} = -R_{CM}\, mg\, sin\theta = I\alpha$$

$$small\ angle\ approximation: sin\theta \approx \theta$$

$$\Longrightarrow \frac{d^2\theta}{dt^2} \approx -\frac{R_{CM}mg}{I}\theta$$

Static Equilibrium

$$\begin{cases} \sum_n \vec{F}_{ext,n} = 0 \\ \sum_m \vec{\tau}_{ext,m} = 0 \end{cases}$$

S.H.M. of a simple pendulum

$$F_{net} = F_t = ma = -mg\ sin\theta$$

$$small\ angle\ approximation: sin\theta \approx \theta$$

$$m\frac{d^2x}{dt^2} \approx -mg\theta = -mg\frac{x}{L}$$

$$\Longrightarrow \frac{d^2x}{dt^2} \approx -\frac{g}{L}x$$

$$or\ m\frac{d^2x}{dt^2} \approx -mg\theta \Rightarrow mL\frac{d^2\theta}{dt^2} \approx -mg\theta$$

$$\Longrightarrow \frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta$$

Static Pressure: $P = P_0 + \rho_{fluid}gh$

The Archimedes Principle:

$$B = \rho_{Fluid}\, V_{Sub}\, g$$

Pascal's Law:

$$P_1 = P_2 \Longleftrightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Continuity Equation:

$$A_1 v_1 = A_2 v_2$$

Bernoulli's Equation:

$$P + \frac{1}{2}\rho_{fluid}v^2 + \rho_{fluid}gy = constant$$

$$1\ atm \equiv 1.013 \times 10^5\ Pa = 760\ mm\ Hg \cong 14.7\ psi$$

$$1\ psi \equiv \frac{1\ lbf}{(1\ inch)^2}$$

$$1\ lb = 0.454\ kg; \ \ 1\ inch = 2.54\ cm$$