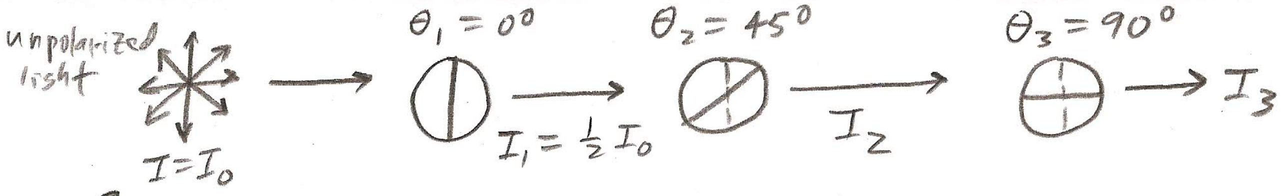


Physics 4C Practice FINAL EXAM

Instructions: There are 10 multiple choice questions (worth 5.5% each) and 3 longer problems (worth 15% each). Read everything *carefully* and give the *best* answer based on the material presented in class and in the text.

(1) Unpolarized light is passed through three successive Polaroid filters, each with its transmission axis at 45° to the preceding filter. What percentage of light gets through? [Hint: the answer is *not* zero, paradoxical as it may sound, because each polarizer after the first rotates the polarization plane.]

a. 6.25% b. 12.5% c. 25% d. 50% e. 33%

Find $- I_3/I_0$

$I_1 = \frac{1}{2} I_0$

$I_2 = I_1 \cos^2(45^\circ - 0^\circ) = I_1 \cos^2(45^\circ)$

$I_3 = I_2 \cos^2(90^\circ - 45^\circ) = I_2 \cos^2(45^\circ)$

$I_3 = I_2 \cos^2(45^\circ)$
$= I_1 \cos^4(45^\circ)$
$= \frac{1}{2} I_0 \cos^4(45^\circ)$

$\boxed{I_3/I_0 = 0.125 \rightarrow \text{choice (b)}}$

(2) An electron has a kinetic energy that is twice its rest energy. Determine its speed.

a. 0.76 c b. 0.81 c c. 0.94 c d. 0.54 c e. 0.87 c

$K = (\gamma - 1) mc^2$
$= 2mc^2$
$(\gamma - 1) = 2$

$\gamma = 3 = 1/\sqrt{1-(v/c)^2}$
$9 = 1/[1-(v/c)^2]$
$(v/c)^2 = 8/9$

$v/c = \sqrt{8}/3 = 2\sqrt{2}/3$

$\boxed{v = 0.943c \rightarrow \text{choice (c)}}$

(3) Two slits separated by 0.10 mm are illuminated with green light ($\lambda = 540$ nm). Calculate the distance (in cm) from the central bright-region to the bright band with $m = 5$ if the screen is 1.0 m away.

a. 2.3 b. 2.5 c. 2.7 d. 2.1 e. 2.0

$d = 0.10\text{mm} = 1.0 \times 10^{-4}\text{m}$
$\lambda = 5.40 \times 10^{-7}\text{m}$
$L = 1.0\text{m}$ $m = 5$ bright band, so constructive.
Find $- Y$

$d \sin\theta = m\lambda$
$dY/L \simeq m\lambda$

$Y = \frac{m\lambda L}{d} = \frac{5(5.40 \times 10^{-7}\text{m})(1.0\text{m})}{(1.0 \times 10^{-4}\text{m})}$

$\boxed{Y = 2.7 \times 10^{-2}\text{m}} = 2.7\text{cm} \rightarrow \text{choice (c)}$

(4) A ruby laser beam ($\lambda = 694.3$ nm) is sent outwards from a 2.7-m diameter telescope to the moon, 384,000 km away. What is the radius of the big red spot on the moon? Assume the telescope has a circular aperture, and neglect the effects of Earth's atmosphere.

a. 750 m b. 500 m c. 120 m d. 1.0 km e. 2.7 km

Diffraction through a circular aperture $-$

$\theta_{min} \simeq \frac{1.22\lambda}{D} = \frac{Y}{L}$ $Y = \frac{(1.22)\lambda L}{D} = \frac{(1.22)(6.943 \times 10^{-7}\text{m})(3.84 \times 10^{8}\text{m})}{2.7\text{m}}$

$\boxed{Y = 120\text{m} \rightarrow \text{Choice (c)}}$

(5) Human body temperature is 98.6° F = 37° C = 310 K . Assuming that human skin is a perfect radiator (with efficiency $\varepsilon = 1$), determine the wavelength corresponding to the largest intensity (in microns, where 1 micron = 1 μm = 10^{-6} m).

a. 8.0 b. 9.3 c. 3.0 d. 5.7 e. 29.4

$T = 310\text{K}$

$\lambda_{max}(\text{m}) = \frac{2.898 \times 10^{-3}\text{m-K}}{310\text{ K}} = \boxed{\lambda_{max} = 9.35 \text{ microns} \rightarrow \text{choice (b)}}$

(6) An electron in a hydrogen atom makes a transition from the $n = 3$ to the $n = 1$ energy state. Determine the wavelength of the emitted photon (in nm).

a. 1006 b. 209 c. 306 d. 103 e. 821

$$\frac{1}{\lambda} = \left(\frac{1}{91.2\,nm}\right)\left[\left(\frac{1}{1^2}\right) - \left(\frac{1}{3^2}\right)\right] = (1/91.2\,nm)\,[1 - (1/9)]$$

$$\lambda = \left(\frac{9}{8}\right)(91.2\,nm) = \boxed{\lambda = 102.6\,nm}$$

$\boxed{\rightarrow \text{Choice (d)}}$

(7) Find the minimum uncertainty in the momentum (in kg · m/s) of an electron if the uncertainty in its position is equal to the circumference of the first Bohr orbit.

a. 6.2×10^{-25}
b. 1.59×10^{-25}
c. 15.5×10^{-25}
d. 19.4×10^{-25}
e. 2.0×10^{-24}

$\Delta x \Delta p_x \geq \hbar/2$ $\qquad c = 2\pi r = 2\pi a_0$

$$\Delta p_x \geq \frac{\hbar}{2(\Delta x)} = \frac{h}{2(2\pi)(2\pi)a_0} = \frac{h}{8\pi^2 a_0}$$

$$= \frac{(6.626\times10^{-34}\,J\text{-}s)}{8\pi^2(5.29\times10^{-11}\,m)} = \boxed{\Delta p_x \geq 1.59\times10^{-25}\,\frac{J\text{-}s}{m}}$$

$\boxed{\rightarrow \text{Choice (b)}}$

(8) The half-life of ^{131}I is 8.04 days. Three days after it was prepared, its activity was 0.50 μCi. How many curies (in μCi) were initially prepared?

a. 0.60 b. 0.70 c. 0.65 d. 0.55 e. 0.39

$N = N_0 e^{-\lambda t}$

$\lambda = \frac{\ln(2)}{T_{1/2}}$

$$\rightarrow \frac{N}{N_0} = \exp\left[-\frac{\ln(2)\,t}{T_{1/2}}\right] \rightarrow N_0 = (0.5\,\mu Ci)\exp\left[\frac{(0.693)(3\,d)}{8.04\,d}\right]$$

Find – N_0

$\boxed{N_0 = 0.65\,\mu Ci} \boxed{\rightarrow \text{Choice (c)}}$

(9) What is the maximum velocity (in km/s) of a photoelectron emitted from a surface with a work function of 5 .0 eV when illuminated by a light whose wavelength is 200 nm?

a. 460 b. 650 c. 420 d. 550 e. 1480

$$= \sqrt{\frac{2(1.20\,eV)}{5.11\times10^5\,eV}}\;c =$$

$$K_{max} = \frac{hc}{\lambda} - \phi = \frac{1}{2} m_e v^2$$

$$\text{So: } v = \sqrt{\frac{2}{m_e}\left(\frac{hc}{\lambda} - \phi\right)} = \sqrt{\left(\frac{2}{5.11\times10^5\,eV/c^2}\right)\left(\frac{1240\,eV\text{-}nm}{200\,nm} - 5\,eV\right)} = \boxed{650\times10^3\,\frac{m}{s}}$$

$\boxed{\rightarrow \text{choice (b)}}$

(10) The magnetic field of a plane-polarized electromagnetic wave moving in the z-direction is given by $B = 1.2 \times 10^{-6} \sin\left[2\pi\left(\frac{z}{240} - \frac{t\times10^7}{8}\right)\right]$ in SI units. Find the average power per square meter carried by the EM wave.

a. 720 W
b. 172 W
c. 500 W
d. 200,000 W
e. 86 W

$$I = \frac{P}{A}\left(\frac{\text{Power}}{\text{area}}\right) = \frac{E_{max} B_{max}}{2\mu_0} = \frac{c B_{max}^2}{2\mu_0},$$

since $\frac{E_{max}}{B_{max}} = c$.

$$\text{So: } I = \frac{(3.0\times10^8\,m/s)(1.2\times10^{-6}\,T)^2}{2(4\pi\times10^{-7}\,Tm/A)}$$

$\boxed{I = 172\,W} \boxed{\rightarrow \text{Choice (b)}}$

Problems. You must explicitly show all your work on this part. No work = no credit. **Box your answers.**

(A) A dedicated sports car enthusiast polishes the inside and outside surfaces of a hubcap that is a section of a sphere.

(a) She holds the hubcap 0.40 m from her face, and looks into the inside surface of the hubcap, which has a radius of curvature of +0.60 m.

(i) She sees her image reflected by the inside surface of the hubcap. What is the image distance of this image?

(ii) What is the magnification of this image?

(iii) Is this a real or a virtual image? Is this image upright or inverted?

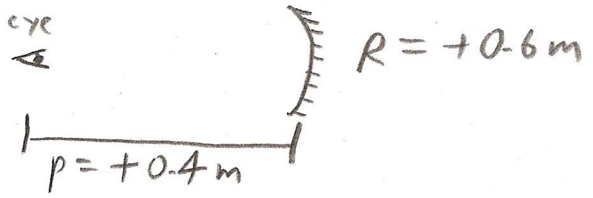
(b) She now flips the hubcap around, so that she looks into the outside surface of the hubcap. Again, she holds the hubcap 0.40 m from her face. The outside surface of the hubcap has a radius of curvature of -0.60 m.

(i) She sees her image reflected by the outside surface of the hubcap. What is the image distance of this image?

(ii) What is the magnification of this image?

(iii) Is this a real or a virtual image? Is this image upright or inverted?

(a)



(i) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$

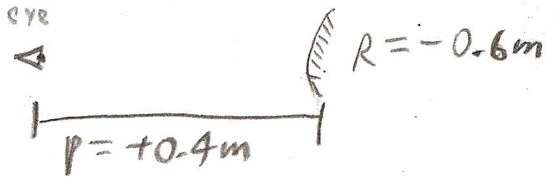
$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{(0.6\,m)} - \frac{1}{(0.4\,m)}$

$\boxed{q = +1.2\,m}$

(ii) $M = -\frac{q}{p} = -\frac{1.2\,m}{0.4\,m} = \boxed{M = -3}$

(iii) This is a real image, since $q > 0$.
This image is inverted, since $M < 0$.

(b)



(i) $\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2}{(-0.6\,m)} - \frac{1}{(0.4\,m)}$

$\boxed{q = -0.17\,m} = -\frac{6}{35}\,m$

(ii) $M = -\frac{q}{p} = \frac{-(-0.17\,m)}{(+0.4\,m)} = \boxed{M = +0.43}$
$= +3/7$

(iii) This is a virtual image, since $q < 0$.
This image is upright, since $M > 0$.

(B) (i) A free particle has no forces on it, so $U(x) = 0$ everywhere ($-\infty < x < +\infty$) for this particle. This particle has a one-dimensional wavefunction $\psi(x) = B\sin kx$, where B is a constant and $k = 2\pi/\lambda$. Use Schrödinger's equation to find the particle's total energy E, as a function of $\hbar$, k, and m.

(ii) Consider a different particle, which is not free, but is in a potential, $U(x)$, which is not equal to zero everywhere. This particle has mass m and a one-dimensional wavefunction given by $\psi(x) = Ke^{-x^2}$, where K is a constant.

If the energy of this particle is given as $E = \frac{\hbar^2}{m}$, use Schrödinger's equation to determine the potential, $U(x)$, for this particle.

(iii) Determine the value of K, from (ii). A definite integral that is useful for this is $\int_{-\infty}^{+\infty} e^{-Ax^2}dx = \sqrt{\pi/A}$, where A is a constant. **Give the answer symbolically, not a numerical answer.**

(i) A free particle has $U(x)=0$ everywhere $(-\infty < x < +\infty)$.

so: $\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E-\cancel{U}^{=0})\psi = -\frac{2m}{\hbar^2}E\psi$

If $\psi = B\sin kx$

then $\frac{d\psi}{dx} = kB\cos kx$

and: $\frac{d^2\psi}{dx^2} = -kB^2\sin kx$

so: $-k^2\cancel{B}\cancel{\sin kx} = -\frac{2m}{\hbar^2}E(\cancel{B}\cancel{\sin kx})$

$+k^2 = +\frac{2m}{\hbar^2}E$

$$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$

(iii) $\int_{-\infty}^{+\infty} |\psi^2(x)|\,dx = 1$, so:

$\int_{-\infty}^{+\infty} K^2 e^{-2x^2}dx = K^2\int_{-\infty}^{+\infty} e^{-Ax^2}dx$ where $A=2$

$= K^2\sqrt{\pi/A} = K^2\sqrt{\pi/2} = 1$

$$\boxed{\text{so: } K = \sqrt[4]{2/\pi}}$$

(ii) $\psi(x) = Ke^{-x^2}$, K constant

Find U, if $E = \frac{\hbar^2}{m}$

$\frac{d\psi}{dx} = -2xKe^{-x^2}$

$\frac{d^2\psi}{dx^2} = -2Ke^{-x^2} - 2xK\frac{d}{dx}e^{-x^2}$

$= -2Ke^{-x^2} - 2xK(-2x)e^{-x^2}$

$= -2Ke^{-x^2} + 4Kx^2e^{-x^2}$

$= -2Ke^{-x^2}[1-2x^2] = -\frac{2m}{\hbar^2}(E-U)\psi = \frac{-2m}{\hbar^2}\left(\frac{\hbar^2}{m} - U\right)Ke^{-x^2}$

$+\frac{\cancel{2}m}{\hbar^2}\left(\frac{\hbar^2}{m} - U\right)\cancel{Ke^{-x^2}} = +\cancel{2}\cancel{Ke^{-x^2}}[1-2x^2]$

$\frac{m}{\hbar^2}\left(\frac{\hbar^2}{m} - U\right) = \left(1 - \frac{mU}{\hbar^2}\right) = 1-2x^2$

$\frac{mU}{\hbar^2} = 2x^2 \Rightarrow \boxed{U = \frac{2\hbar^2}{m}x^2 = 2Ex^2}$

(C) (i) In a television tube, electrons are accelerated through a potential difference of 2,500,000 (2.5×10^6) volts. At what speed do the electrons strike the screen? Importantly, <u>express your answer as a decimal fraction of *c*</u>, the speed of light in empty space (for example, half the speed of light in empty space is 0.5 *c*).

[*Hints*: remember that 1 electron-volt = 1 eV = 1.602×10^{-19} Joules, which is the amount of energy one electron gets when it is accelerated through a potential difference of one volt, and that $U = qV$, where U = potential energy, q = charge, and V = electrostatic potential.]

$$(1C)(1V) = 1J$$
$$(1.602\times10^{-19}C)(1V) = 1.602\times10^{-19}J \equiv 1eV$$
$$(1.602\times10^{-19}C)(2.5\times10^6V) = 2.5\times10^6 eV = K$$
$$2.5\times10^6 eV = K = (\gamma-1)m_e c^2, \quad m_e = 0.511\ MeV/c^2$$
$$\frac{(2.5\times10^6 eV)}{(0.511\times10^6 \frac{eV}{c^2})c^2} = \gamma - 1$$
$$\frac{2.5}{0.511} + 1 = \gamma = 5.892 = \frac{1}{\sqrt{1-(v/c)^2}}$$
$$34.72 = 1/[1-(v/c)^2]$$
$$1-(v/c)^2 = 1/(34.72)$$
$$\frac{v}{c} = \sqrt{1-\left(\frac{1}{34.72}\right)}$$
$$\boxed{v = 0.986c}$$

(ii) Show that any electron orbiting in any energy level in a hydrogen atom must be moving at non-relativistic speeds (in other words, with $v < 0.1\ c$). [*Hint:* this can be done by calculating the speed of an electron in the $n = 1$ orbit of a hydrogen atom.]

An electron in a H atom has:

$$m_e v r = n\hbar,$$
$$\text{so: } v = \frac{n\hbar}{m_e a_0 n^2} = \frac{h}{2\pi m_e a_0 n^2} = \frac{6.626\times10^{-34} J\cdot s}{(2\pi)(9.11\times10^{-31}kg)(5.29\times10^{-11}m)(1)^2}$$
(since $r = a_0 n^2$)
$$\boxed{v = 2.19\times10^6 m/s = 7.3\times10^{-3}c \simeq \left(\frac{1}{137}\right)c \ll 0.1c.}$$

(iii) After a living thing dies, the ^{14}C in it decays to ^{12}C with a half-life of 5730 years. Suppose an archaeologist finds an ancient firepit containing some partially consumed firewood. This wood contains only 12.5 percent of the ^{14}C content of an equal carbon sample from a present-day tree. What is the age of the ancient firewood?

$$N = N_0 e^{-\lambda t} = N_0 \exp\left[-\frac{0.693\, t}{\tau_{1/2}}\right]$$
$$\text{so: } t = \frac{-(\tau_{1/2})\ln(N/N_0)}{0.693} = \frac{-(5730\text{ years})\ln(0.125)}{0.693}$$
$$\boxed{t = 17,200 \text{ years}}$$