Phys 4C Practice Final Exam

(1) $\lambda = 410\,nm = 4.10 \times 10^{-7}\,m$

$d = 0.093\,mm = 9.3 \times 10^{-5}\,m$

$L = 2.8\,m$

Find Δy for two-slit diffraction.

$d \sin\theta = m\lambda$

$\frac{dY}{L} \simeq m\lambda$

$Y \simeq \frac{m\lambda L}{d}$

$\Delta y = Y_4 - Y_1 \simeq (4-1)\frac{\lambda L}{d} = \frac{3\lambda L}{d}$

$= \frac{3(4.10 \times 10^{-7}\,m)(2.8\,m)}{(9.3 \times 10^{-5}\,m)}$

$\boxed{\Delta y \simeq 0.037\,m}$ $\boxed{\Rightarrow \text{choice (d)}}$

(2) $K = qV = 1000\,eV = \frac{1}{2} m_e v^2$

Find $\lambda = \frac{h}{p}$.

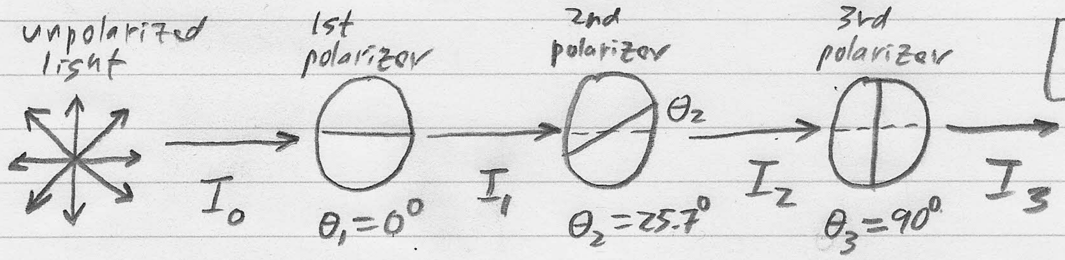
$2Km_e = (m_e v)^2$

$\sqrt{2Km_e} = m_e v = p$

$\lambda = \frac{h}{\sqrt{2Km_e}} = \frac{6.626 \times 10^{-34}\,J\cdot s}{\sqrt{(2)(1000\,eV)\left(\frac{1.609 \times 10^{-19}\,J}{1\,eV}\right)(9.11 \times 10^{-31}\,kg)}}$

$\boxed{\lambda = 3.9 \times 10^{-11}\,m}$ $\boxed{\Rightarrow \text{choice (e), none of the above}}$

(3)



Find I_3/I_0

$I_1/I_0 = 0.5$ since I_0 is unpolarized light.

Malus's law is $I_2/I_1 = \cos^2(\theta_2 - \theta_1)$,

so: $I_2/I_1 = \cos^2(25.7°)$

$I_3/I_2 = \cos^2(90° - 25.7°)$

$\Rightarrow I_3/I_0 = 0.5\cos^2(25.7°)\cos^2(90° - 25.7°)$

$\boxed{I_3/I_0 = 0.076 = 7.6\%}$ $\boxed{\Rightarrow \text{choice (a)}}$

(4) $a \sin\theta = m\lambda$, $m = \pm 1, \pm 2, \pm 3 \ldots$

$a_1 \sin\theta_1 = a_2 \sin\theta_2$

Since $\sin\theta \simeq \theta$ for small θ, θ in radians,

$a_1\theta_1 \simeq a_2\theta_2$

Doubling a will cause the separation of the minima to decrease by 1/2

$\boxed{\Rightarrow \text{Choice (d)}}$

$\boxed{\text{for Serway's text}}$

$\boxed{\text{Find} - p}$

(5) A concave mirror has $f > 0$ and $R > 0$.
Here, $f = +35.0$ cm.
The image is upright, so $M > 0$.
The image is 5 times the size of the object,
so $M = +5 = -\frac{q}{p}$ and so $q = -5p$.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{p} - \frac{1}{5p} = \frac{1}{35.0\text{ cm}}.$$

$$\frac{5-1}{5p} = \frac{4}{5p} = \frac{1}{35.0\text{cm}} \quad \therefore \boxed{p = +28.0\text{ cm} \;\Rightarrow \text{choice (c)}}$$

(6) $m_1 = 3.0\times10^{-28}$ kg
$v_1 = 0.793\,c$ → For this, we need relativistic momentum, $p \equiv \gamma m v$.
(For classical physics at speeds $v \lll c$, $p = mv$ is OK, but it isn't, here.)

$m_2 = 1.67\times10^{-27}$ kg
$\boxed{\text{Find} - v_2}$

By conservation of relativistic momentum,

$$\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$$

$$\frac{m_1 v_1}{\sqrt{1-(v_1/c)^2}} = \frac{m_2 v_2}{\sqrt{1-(v_2/c)^2}}$$

$$\frac{m_1^2 v_1^2}{1-(v_1/c)^2} = \frac{m_2^2 v_2^2}{1-(v_2/c)^2} \rightarrow \left(\frac{m_1}{m_2}\right)^2\left[\frac{1}{(c/v_1)^2-1}\right] = \left[\frac{1}{(c/v_2)^2-1}\right]$$

$$\Rightarrow \boxed{v_2 = 0.228\,c \;\Rightarrow \text{choice (a)}}$$

(7) $\boxed{\text{Find} - \Delta p_x}$

Check units — $1\text{ J} = 1\text{ kg m}^2/\text{s}^2$
$1\,\frac{\text{J}\cdot\text{s}}{\text{m}} = 1\text{ kg m/s}$ ✓

$\Delta x = 0.5\times10^{-9}$ m.
Since $\Delta x \Delta p_x \gtrsim \hbar/2$,

$$\Delta p_x \gtrsim \frac{\hbar}{2(\Delta x)} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{(2)(2\pi)(0.5\times10^{-9}\text{ m})} = \boxed{1.1\times10^{-25}\,\frac{\text{J}\cdot\text{s}}{\text{m}} \;\Rightarrow \text{choice (a)}}$$

for Serway's text

(8) Both forbidden transitions and selection rules exist because $\Delta \ell = \pm 1$ for electrons making transitions in atoms, which radiate (or absorb) photons. During a transition, the electron changes its angular momentum, since $\Delta \ell = \pm 1$. It also radiates (or absorbs) a photon during the transition. Since angular momentum is conserved, this implies that a photon must have angular momentum.
$\Rightarrow$ $\boxed{\text{choice (c)}}$

(9) $\tau_{1/2} = 8.04\,d$
$t = 3\,d$
$R = 0.5\,\mu Ci$
$\boxed{\text{Find} - R_0}$

$N = N_0 e^{-\lambda t}$ and $R = dN/dt$,

so: $R_0 = R e^{\lambda t}$

$= R e^{(\ln 2) t / \tau_{1/2}}$

$= (0.5\,\mu Ci) \exp\left[\frac{(0.693)(3d)}{(8.04d)}\right]$

$\boxed{R_0 = 0.65\,\mu Ci}$ $\boxed{\Rightarrow \text{choice (d)}}$

(10) $I = S_{av} = 1340\,\frac{W}{m^2} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c}$, since $\frac{E_{max}}{B_{max}} = \frac{E}{B} = c$.

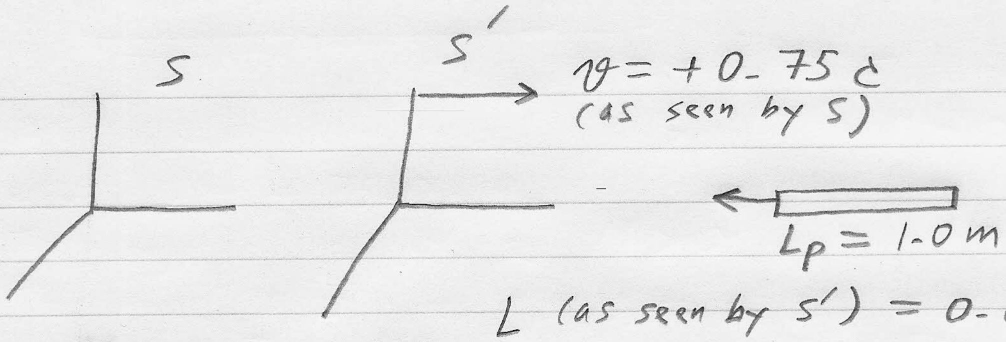
$\Rightarrow E_{max} = \sqrt{2\mu_0 c I}$

$= \left[(2)(4\pi \times 10^{-7}\,\frac{Tm}{A})(3.0\times 10^{8}\,\frac{m}{s})(1340\,\frac{W}{m^2})\right]^{1/2}$

$\boxed{E_{max} = 1000\,\frac{V}{m}}$ $\boxed{\Rightarrow \text{choice (b)}}$

check units: $\left(\frac{T\cancel{m}}{A}\right)\left(\frac{\cancel{m}}{s}\right)\left(\frac{W}{\cancel{m^2}}\right) = \frac{TW}{As} = \left(\frac{V}{m^2}\right)\left(\frac{\cancel{s}}{C}\right)\left(\frac{1}{\cancel{s}}\right) = \frac{V^2}{m^2}$ ✓

(A)



L (as seen by S') $= 0.6\,m$

(i) As seen by S',

$$L = \frac{L_p}{\gamma} = 0.6m = \frac{1.0\,m}{\left(1/\sqrt{1-(u_x'/c)^2}\right)} = 1.0m\sqrt{1-(u_x'/c)^2}$$

$$\left(\frac{0.6}{1.0}\right)^2 = 1 - \left(\frac{u_x'}{c}\right)^2$$

$$(u_x'/c) = \sqrt{1-(0.6)^2} \qquad \therefore \boxed{u_x' = -0.8c}$$

as seen by S'.

This is negative since this stick is going <u>left</u>, whereas frame S' is going <u>right</u>.

(ii) As seen by S,

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{-0.8c + 0.75c}{1 + \frac{(-0.8\cancel{c})(0.75\cancel{c})}{\cancel{c^2}}}$$

$$= \frac{-0.05c}{1-[(0.8)(0.75)]} = \frac{-0.05c}{0.4}$$

$$\therefore \boxed{u_x = -0.125c}$$

(iii) As seen by S,

$$L = \frac{L_p}{\gamma} = (1.0m)\sqrt{1-(u_x/c)^2}$$

$$= (1.0m)\sqrt{1-(-0.125)^2}$$

$$\boxed{L = 0.992\,m}$$

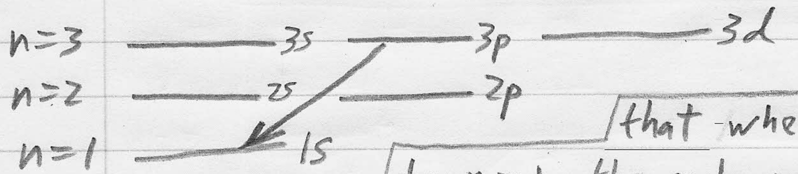
(i) For $n=3$, ℓ can have the values $\ell = 0, 1, 2, \ldots, n-1$,

(B) so $\boxed{\ell = 0, 1, \text{ or } 2}$.

m_ℓ can have the values $m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell$,

so $\boxed{m_\ell = -2, -1, 0, 1, \text{ or } 2}$.

(ii) $\ell=0$ $\quad \ell=1 \quad \ell=2$



Because the atom emits a photon during an electron transition, $\Delta \ell = \pm 1$. This means that when an electron jumps from $n=3$ to $n=1$, the only way it can do so is to go from $\ell=1$ to $\ell=0$, as shown at left.

Also, for $n=1$, $\ell=0$ only, so $m_\ell = 0$ only when $n=1$.
Since $\Delta m_\ell = 0$ or $+1$ or -1 only, the only permitted transitions for an electron going from $n=3$ to $n=1$ have:

$$\boxed{\begin{array}{ll} \ell=1 \text{ and } m_\ell = 0 \\ \ell=1 \text{ and } m_\ell = +1 \\ \ell=1 \text{ and } m_\ell = -1 \end{array}}$$

(iii) $$p = \int p(x)\,dx = \int |\psi(x)|^2 dx = \int_{x=0}^{x=L/3} \left(\frac{2}{L}\right) \sin^2\left(\frac{\pi x}{L}\right) dx$$

$$= \left(\frac{2}{L}\right) \int_0^{L/3} \sin^2 ax\, dx, \text{ where } a = \pi/L$$

Since $\int \sin^2 (ax)\, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$,

$$p = \left(\frac{2}{L}\right)\left(\frac{x}{2} - \frac{\sin(2ax)}{4a}\right\Big|_{x=0}^{x=L/3} = \frac{2}{L}\left(\frac{x}{2} - \frac{L\sin\left(\frac{2\pi x}{L}\right)}{4\pi}\right|_{x=0}^{x=L/3}$$

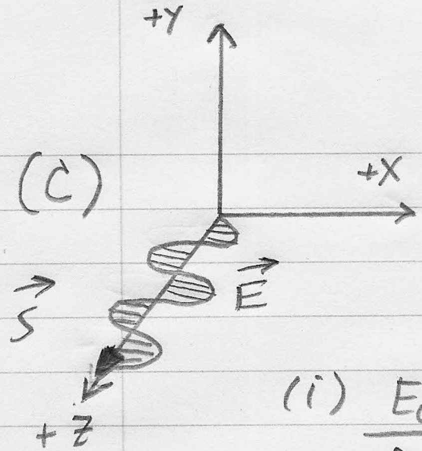
$$= \left(\frac{2}{L}\right)\left(\frac{L}{2(3)} - \frac{L\sin(2\pi/3)}{4\pi} - 0 - 0\right)$$

$\therefore \boxed{p = 0.1955}$

(iv) $$\int_{x=-\infty}^{+\infty} |\psi(x)|^2 dx = 1 = \int_{x=-L}^{x=+L} |A|^2 dx, \text{ so } A^2 x \Big|_{-L}^{+L} = 1,$$

and: $L - (-L) = 2L = 1/A^2$, so $\boxed{A = \frac{1}{\sqrt{2L}}}$.

(C)



$\lambda = 3.50\,cm = 3.5\times10^{-2}\,m$

$E_0 = 275\ V/m$

$B = B_0 \sin(kx - wt)$

(i) $\frac{E_0}{B_0} = c \Rightarrow B_0 = \frac{E_0}{c} = \frac{275\ V/m}{3.0\times10^{8}\ m/s}$

$\boxed{B_0 = 9.2\times10^{-7}\,T}$

check units: $\frac{V}{m} = \frac{Tm}{s}$ ✓

The $\vec{B}$ field is along the +Y direction,

since $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$.

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.5\times10^{-2}m} = \boxed{k = 180\ m^{-1}}$.

$w = 2\pi f = \frac{2\pi c}{\lambda} = kc = \boxed{w = 5.39\times10^{10}\ s^{-1}.}$

(ii) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\left[E_0\sin(kx-wt)\hat{x}\right]\times\left[B_0\sin(kx-wt)\hat{y}\right]$

$= \left(\frac{E_0B_0}{\mu_0}\right)\left[\sin^2(kx-wt)\right]\hat{x}\times\hat{y}$

$= \left(\frac{(275\frac{V}{m})(9.2\times10^{-7}T)}{4\pi\times10^{-7}\ Tm/A}\right)\sin^2(kx-wt)\,\hat{z}$

$\boxed{\vec{S} = \left(200\frac{W}{m^2}\right)\left[\sin^2(kx-wt)\right]\hat{z}}$

Check units:
$W = Fd$
$J = Nm$
$W = \frac{Nm}{s}$
$\frac{V}{m}\,\frac{\cancel{T}}{\cancel{T}}\,\frac{A}{m} = \frac{N\cancel{C}}{\cancel{C}\,sm} = \frac{W}{m^2}$ ✓
$\frac{W}{m^2}\,\frac{s}{m} = \frac{Ws}{m^3} = \frac{N}{m^2}$ ✓

Averaged over one cycle, this is:

$I = S_{av} = \frac{E_0B_0}{2\mu_0} = 100\frac{W}{m^2}$, $\boxed{\text{in the +z direction}}$

(iii) For complete reflection,

$p = \frac{2I}{c} = \boxed{p = 6.67\times10^{-7}\ N/m^2}$.

(iv) Neon $(z=10)$ has a ground-state electronic configuration of: $\boxed{1s^2\,2s^2\,2p^6}$.