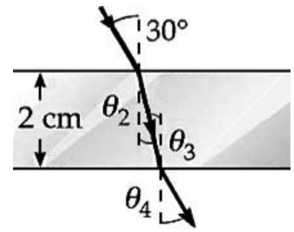
P34.10 At entry, the wave under refraction model, expressed as

$n_1 \sin\theta_1 = n_2 \sin\theta_2$, gives

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.000 \sin 30.0°}{1.50}\right) = \boxed{19.5°}$$



ANS. FIG. P34.10

To do ray optics, you must remember some geometry. The surfaces of entry and exit are parallel so their normals are parallel. Then angle θ_2 of refraction at entry and the angle θ_3 of incidence at exit are alternate interior angles formed by the ray as a transversal cutting parallel lines.

Therefore, $\theta_3 = \theta_2 = \boxed{19.5°}$.

At the exit point, $n_2 \sin\theta_3 = n_1 \sin\theta_4$ gives

$$\theta_4 = \sin^{-1}\left(\frac{n_2 \sin\theta_3}{n_1}\right) = \sin^{-1}\left(\frac{1.50 \sin 19.5°}{1.000}\right) = \boxed{30.0°}$$

Because θ_1 and θ_4 are equal, the departing ray in air is parallel to the original ray.

P34.18 Note for use in every part (refer to ANS. FIG. P34.18): from apex angle F,

$$\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$$

so $\quad \theta_3 = \Phi - \theta_2$

At the first surface the deviation is

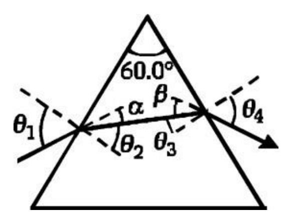
$$\alpha = \theta_1 - \theta_2$$

At exit, the deviation is

$$\beta = \theta_4 - \theta_3$$

The total deviation is therefore

$$\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$$



ANS. FIG. P34.18

(a) At entry,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{\sin 48.6°}{1.50}\right) = 30.0°$$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit,

$$1.50 \sin 30.0° = 1.00 \sin\theta_4$$

or $\quad \theta_4 = \sin^{-1}\left[1.50 \sin(30.0°)\right] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry,

$$\sin\theta_2 = \frac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$$

$$\theta_3 = 60.0° - 28.4° = 31.6°$$

At exit,

$$\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$$

$$\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$$

(d) At entry,

$$\sin\theta_2 = \frac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$$

$$\theta_3 = 60.0° - 31.5° = 28.5°$$

At exit,

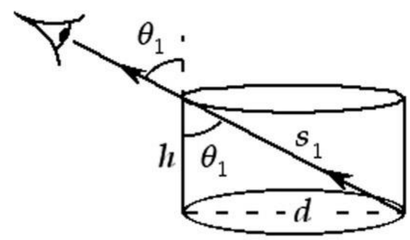
$$\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$$

$$\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$$

P34.20 (a) Before the container is filled, the ray's path is as shown in ANS. FIG. P34.20 (a). From this figure, observe that

$$\sin\theta_1 = \frac{d}{s_1} = \frac{d}{\sqrt{h^2 + d^2}}$$

$$= \frac{1}{\sqrt{(h/d)^2 + 1}}$$

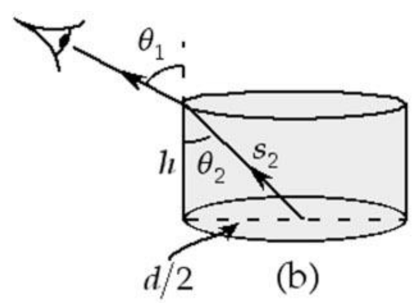


ANS. FIG. P34.20 (a)

After the container is filled, the ray's path is shown in ANS. FIG. P34.20 (b). From this figure, we find that

$$\sin\theta_2 = \frac{d/2}{s_2} = \frac{d/2}{\sqrt{h^2 + (d/2)^2}}$$

$$= \frac{1}{\sqrt{4(h/d)^2 + 1}}$$



ANS. FIG. P34.20(b)

From Snell's law, we have

$$1.00\sin\theta_1 = n\sin\theta_2$$

$$\frac{1.00}{\sqrt{(h/d)^2+1}} = \frac{n}{\sqrt{4(h/d)^2+1}}$$

$$4(h/d)^2+1 = n^2(h/d)^2+n^2$$

$$(h/d)^2(4-n^2) = n^2-1 \rightarrow \boxed{\frac{h}{d} = \sqrt{\frac{n^2-1}{4-n^2}}}$$

(b) For water, $n = 1.333$.

$$\frac{h}{d} = \sqrt{\frac{n^2-1}{4-n^2}}$$

$$\frac{h}{8.00\text{ cm}} = \sqrt{\frac{(1.333)^2-1}{4-(1.333)^2}} = \boxed{4.73\text{ cm}}$$

(c) $\boxed{\text{For } n=1,\ h=0.\ \text{For } n=2,\ h=\infty.\ \text{For } n>2,\ h \text{ has no real solution.}}$

P34.23 The reflected ray and refracted ray are perpendicular to each other, and the angle of reflection θ_1 and the angle of refraction θ_2 are related by

$$\theta_1 + 90.0° + \theta_2 = 180.0° \rightarrow \theta_2 = 90.0° - \theta_1$$

Then, from Snell's law,

$$\sin\theta_1 = \frac{n_g \sin\theta_2}{n_{\text{air}}}$$

$$= n_g \sin(90° - \theta_1) = n_g \cos\theta_1$$

Thus, $\dfrac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = n_g$ or $\boxed{\theta_1 = \tan^{-1}(n_g)}$

P34.24 Using Snell's law gives

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.455}\right)$$

and $\theta_{\text{violet}} = \sin^{-1}\left(\dfrac{n_{\text{air}}\sin\theta_i}{n_{\text{violet}}}\right) = \sin^{-1}\left(\dfrac{(1.000)\sin 50.00°}{1.468}\right)$

Thus, the dispersion is $\theta_{\text{red}} - \theta_{\text{violet}} = \boxed{0.314°}$

P34.27 From Equation 34.9, $\sin\theta_c = \frac{n_2}{n_1}$, where $n_2 = 1.000\ 293$. Values for n_1 come from Table 34.1,

(a) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{2.20}\right) = \boxed{27.0°}$

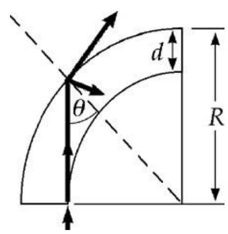
(b) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{1.66}\right) = \boxed{37.1°}$

(c) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{1.309}\right) = \boxed{49.8°}$

P34.31 (a) If any ray escapes it will be a ray along the inner edge, because it has the smallest angle of incidence. Its angle of incidence is described by $\sin\theta = \frac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

$$\frac{n(R-d)}{R} > 1 \quad \rightarrow \quad nR - nd > R$$

$$\rightarrow \quad nR - R > nd \quad \rightarrow \quad R > \boxed{\frac{nd}{n-1}}$$



ANS. FIG. P34.31

(b) As $d \rightarrow 0$, $\boxed{R_{\min} \rightarrow 0}$. $\boxed{\text{Yes: for very small } d\text{, the light strikes the interface at very large angles of incidence.}}$

(c) As n increases, $\boxed{R_{\min} \text{ decreases. Yes: as } n \text{ increases, the critical angle becomes smaller.}}$

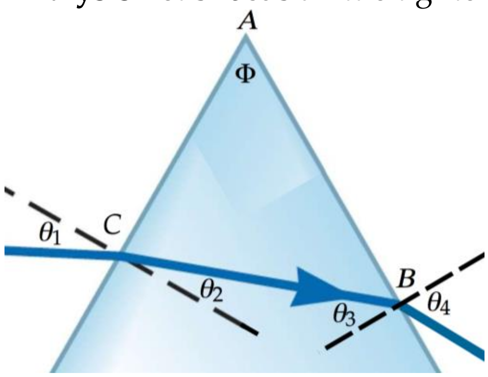
(d) As n decreases toward 1, $R_{\min}$ increases. $\boxed{R_{\min} \rightarrow \infty\text{. Yes: as } n \rightarrow 1\text{, the critical angle becomes close to 90° and any bend will allow the light to escape.}}$

(e) $R_{\min} = \frac{1.40\left(100 \times 10^{-6}\ \text{m}\right)}{0.40} = 350 \times 10^{-6}\ \text{m} = \boxed{350\ \mu\text{m}}$

***P34.34** **Conceptualize** At each surface, the angles of incidence and refraction must satisfy Snell's law. Do you think we'll need Snell's law to solve this problem?

Categorize This problem involve refraction of light in a prism, so we will expect to use the *waves under refraction* analysis model.

Analysis Let's focus a little tighter on the sites of refraction:



We have added three labels for points *A*, *B*, and *C*. Consider now the triangle *ABC*. The sums of the three angles in the triangle must add to 180°:

$$\Phi+\left(90^\circ-\theta_2\right)+\left(90^\circ-\theta_3\right)=180^\circ \quad \rightarrow \quad \Phi=\theta_2+\theta_3$$

which is what we set out to prove.

Finalize Did we ever use Snell's law? No! This suggests that what we have shown is a general geometric result.

Answer: See solution

P34.36 The number *N* of reflections the beam makes before exiting at the other end is equal to the length of the slab divided by the component of the displacement of the beam for each reflection:

$$N=\frac{L}{\left(t/\tan\theta_2\right)}=\frac{L\tan\theta_2}{t}$$

where θ_2 is the refracted angle as the beam enters the material. Substitute for this refracted angle in terms of the incident angle by using Snell's law:

$$N=\frac{L}{t}\tan\left[\sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)\right]$$

Substitute numerical values:

$$N=\frac{0.420\text{ m}}{0.003\ 10\text{ m}}\tan\left[\sin^{-1}\left(\frac{(1)\sin 50.0^\circ}{1.48}\right)\right]$$

$$=81.96 \rightarrow 81\text{ reflections}$$

Therefore, the beam will exit after making 81 reflections, so it does not make 85 reflections.

P34.41 From Table 34.1, the index of refraction of polystyrene is 1.49.

(a) For polystyrene *surrounded by air*, total internal reflection requires

$$\theta_3 \geq \theta_c = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

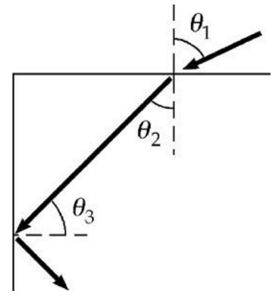
Then from geometry, $\theta_2 = 90.0° - \theta_3 \leq 47.8°$.

From Snell's law,

$$\sin\theta_1 = 1.49\sin\theta_2 \leq 1.49\sin 47.8°$$

$$\sin\theta_1 \leq 1.10$$

Any angle θ_1 satisfies this equation.



ANS. FIG. P34.41

$\boxed{\text{Total internal reflection occurs for all values of } \theta, \text{ or the maximum angle is } 90°.}$

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$.

From Snell's law, $\theta_1 = \boxed{30.3°}$.

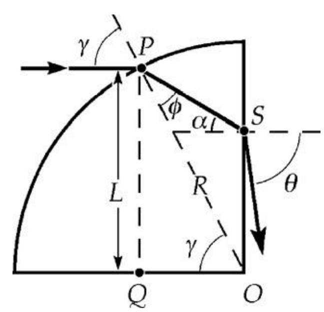
(c) From Table 34.1, the index of carbon disulfide is $1.628 > 1.49$.

$\boxed{\text{Total internal reflection never occurs as the light moves from lower-index polystyrene into higher-index carbon disulfide.}}$

P34.43 Observe in ANS. FIG. P34.43 that the angle of incidence at point P is γ, and using triangle OPQ:

$$\sin\gamma = \frac{L}{R}$$

Also, $$\cos\gamma = \sqrt{1-\sin^2\gamma} = \frac{\sqrt{R^2 - L^2}}{R}$$



ANS. FIG. P34.43

Apply Snell's law at point P:

$$1.00\sin\gamma = n\sin\phi$$

Thus, $$\sin\phi = \frac{\sin\gamma}{n} = \frac{L}{nR}$$

and $$\cos\phi = \sqrt{1-\sin^2\phi} = \frac{\sqrt{n^2R^2 - L^2}}{nR}.$$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$, or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S

gives $\quad 1.00\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$

or
$$\sin\theta = n\sin(\gamma-\phi)$$
$$= n[\sin\gamma\cos\phi - \cos\gamma\sin\phi]$$
$$= n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$
$$= \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)$$

thus,
$$\theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]};$$

or, using from above $\sin\gamma = \frac{L}{R} \to \gamma = \sin^{-1}\frac{L}{R}$ and $\phi = \sin^{-1}\frac{L}{nR}$,
$$\sin\theta = n\sin(\gamma-\phi) = n\sin\left(\sin^{-1}\frac{L}{R} - \sin^{-1}\frac{L}{nR}\right)$$
$$\theta = \boxed{\sin^{-1}\left[n\sin\left(\sin^{-1}\frac{L}{R} - \sin^{-1}\frac{L}{nR}\right)\right]}$$

P34.54 (a) In the textbook Figure P34.54, we have $r_1 = \sqrt{a^2+x^2}$ and $r_2 = \sqrt{b^2+(d-x)^2}$. The speeds in the two media are $v_1 = c/n_1$ and $v_2 = c/n_2$ so the travel time for the light from P to Q is indeed
$$\Delta t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2+x^2}}{c} + \frac{n_2\sqrt{b^2+(d-x)^2}}{c}$$

(b) Now $\frac{d(\Delta t)}{dx} = \frac{n_1}{2c}\frac{2x}{\sqrt{a^2+x^2}} + \frac{n_2}{2c}\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}} = 0$ is the requirement for minimal travel time, which simplifies to
$$\frac{n_1 x}{\sqrt{a^2+x^2}} = \frac{n_2(d-x)}{\sqrt{b^2+(d-x)^2}}$$

(c) Now $\sin\theta_1 = \frac{x}{\sqrt{a^2+x^2}}$ and $\sin\theta_2 = \frac{d-x}{\sqrt{b^2+(d-x)^2}}$, so we have $n_1\sin\theta_1 = n_2\sin\theta_2$.