P35.10 (a) Since the mirror is concave, $R > 0$, giving $f = \frac{R}{2} = +12.0\text{ cm}$. The magnification is positive because the image is upright:

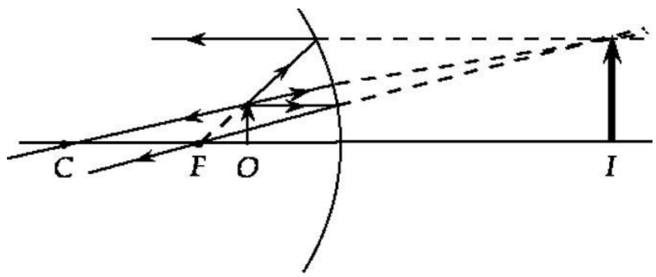
$$M = -\frac{q}{p} = +3 \quad \rightarrow \quad q = -3p$$

The mirror equation is then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} - \frac{1}{3p} = \frac{2}{3p} = \frac{1}{12.0\text{ cm}} \quad \rightarrow \quad p = \boxed{8.00\text{ cm}}$$

(b) ANS. FIG. P35.10(b) shows the principal ray diagram for this situation.



ANS. FIG. P35.10 (b)

(c) The image distance is negative, so the image is $\boxed{\text{virtual}}$. The rays of light do not actually come from the position of the image.

P35.13 The ball is a convex mirror with a diameter of 8.50 cm:

$$R = -4.25\text{ cm} \quad \text{and} \quad f = \frac{R}{2} = -2.125\text{ cm}$$

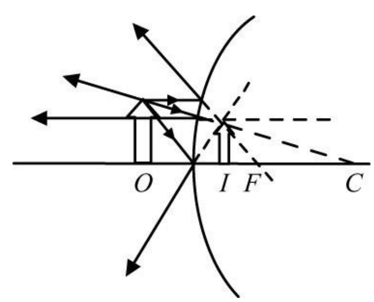
(a) We have

$$M = \frac{3}{4} = -\frac{q}{p} \quad \rightarrow \quad q = -\frac{3}{4}p$$

By the mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125\text{ cm}}$$



ANS. FIG. P35.13

or $\quad \dfrac{3}{3p} - \dfrac{4}{3p} = \dfrac{1}{-2.125\text{ cm}} = \dfrac{-1}{3p} \quad \rightarrow \quad p = +0.708\text{ m}$

The object is $\boxed{\text{0.708 m in front of the sphere}}$.

(b) From ANS. FIG. P35.13, the image is $\boxed{\text{upright}}$, virtual, and diminished.

P35.16 (a) We assume the object is real; thus the object distance p is positive. The mirror is convex, so it is a diverging mirror, and we have $f = -|f| = -8.00\text{ cm}$. The image is virtual, so $q = -|q|$. Since we also know that $|q| = p/3$, the mirror equation gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{3}{p} = \frac{1}{f} \quad \text{or} \quad -\frac{2}{p} = \frac{1}{-8.00\text{ cm}}$$

so $\quad p = +16.0\text{ cm}$

This means that the object is $\boxed{\text{16.0 cm from the mirror}}$.

(b) The magnification is $M = -q/p = +|q|/p = +1/3 = \boxed{+0.333}$.

(c) Thus, the image is $\boxed{\text{upright}}$ and one-third the size of the object.

P35.18 Since the center of curvature of the surface is on the side the light comes from, $R < 0$ giving $R = -4.00$ cm. For the line, $p = 4.00$ cm; then,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes

$$\frac{1.00}{q} = \frac{1.00 - 1.50}{-4.00\text{ cm}} - \frac{1.50}{4.00\text{ cm}}$$

or $\quad q = -4.00\text{ cm}$

Thus, the magnification $M = \dfrac{h'}{h} = -\left(\dfrac{n_1}{n_2}\right)\dfrac{q}{p}$ gives

$$h' = -\left(\frac{n_1 q}{n_2 p}\right)h = -\frac{1.50(-4.00\text{ cm})}{1.00(4.00\text{ cm})}(2.50\text{ mm}) = \boxed{3.75\text{ mm}}$$

***P35.21** **Conceptualize** In this problem, we use the discussion in Section 35.3 regarding images formed by refracting surfaces. In fact, Figure 35.20 applies here if the plastic sphere is changed to a larger water sphere and the coin to a fish.

Categorize We model the light waves leaving the fish as *waves under refraction.*

Analyze (a) The wave under refraction model has given us Equation 35.9 relating the image and object positions for the fish. Solve this equation for the ratio q/p and use Equation 35.10 to find the magnification of the image, substituting the radius as $-R$ because the center of curvature of the front surface of the bowl is in back of the surface.:

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{-R} \quad \rightarrow \quad \frac{q}{p}=-\frac{n_2 R}{(n_2-n_1)p+n_1 R} \quad (1)$$

$$\rightarrow \quad M=-\frac{n_1}{n_2}\frac{q}{p}=\frac{n_1 R}{(n_2-n_1)p+n_1 R} \quad (2)$$

The largest value of p occurs when the fish is on the far side of the tank at $p = 2R$:

$$M_{p=2R}=-\frac{n_1 R}{(n_2-n_1)(2R)+n_1 R}=\frac{n_1}{2n_2-n_1}$$

The smallest value of p occurs when the fish is right up against the glass on the near side of the bowl, $p = 0$:

$$M_{p=0}=\frac{n_1 R}{(n_2-n_1)(0)+n_1 R}=1$$

Therefore, the range of magnifications of the image of the fish is

$$1<M<\frac{n_1}{2n_2-n_1}$$

where we have not used the ≤ sign because the fish is not actually a particle and cannot place his entire body at $p = 2R$ or $p = 0$. Substitute numerical values:

$$1<M<\frac{1.33}{2(1.00)-1.33} \quad \rightarrow \quad \boxed{1<M<1.99}$$

(b) Now, what about your roommate's concern that the Sun's rays will focus on the fish? In this part of the problem, we are looking at the refraction of the Sun's rays as they enter the refracting surface at the *back* of the bowl. From Equation (1), find the image position for the Sun due to the curved surface of the water on which the Sun's rays shine, with the radius of curvature as +*R*:

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R} \quad \rightarrow \quad q=\frac{n_2 R p}{(n_2-n_1)p-n_1 R} \qquad (3)$$

Because the Sun is so far away and its rays arriving at the fishbowl are parallel, the object distance p is essentially infinite and the term $n_1 R$ in the denominator can be neglected. Therefore,

$$q \rightarrow \frac{n_2 R p}{(n_2-n_1)p}=\frac{n_2 R}{(n_2-n_1)} \qquad (4)$$

Substitute numerical values, keeping in mind that the Sun's rays originate outside the bowl:

$$q=\frac{(1.33)R}{(1.33-1.00)}=4.03R \qquad (5)$$

Equation (5) tells us that the focal point of the Sun's rays for the first refracting surface is beyond the opposite surface of the bowl, which is a distance 2*R* away. (The second surface will also refract the Sun's rays, but the overall focal point will still be outside the bowl.) Therefore, we do not have to worry about the fish swimming through the focal point of the Sun's rays.

Finalize With regard to this last point, the fish bowl will focus the Sun's rays on a point inside the room near the fishbowl, so you may want to make sure nothing flammable is located at that point! If the fishbowl is sitting on a wooden table in the Sun, beware! (Check YouTube!)

Answers: (a) $1.00 < M < 1.99$ (b) No; the light from the Sun does not focus within the bowl.

***P35.22 Conceptualize** In this problem, we use the discussion in Section 35.3 regarding images formed by refracting surfaces. The Sun's rays will refract once upon entering the sphere and again upon exiting.

Categorize We model the light waves from the Sun encountering the sphere as *waves under refraction*.

Analyze The wave under refraction model has given us Equation 35.9 relating the image and object positions for a refracting surface. Solve this equation for the image position:

$$\frac{n_1}{p_a} + \frac{n_2}{q_a} = \frac{n_2 - n_1}{R} \quad \rightarrow \quad q_a = \frac{n_2 R p_a}{\left(n_2 - n_1\right)p_a - n_1 R} \quad (1)$$

where n_1 is for material surrounding the sphere (air in our example) and n_2 is for the glass from which the sphere is made. We have also used the subscript *a* to represent quantities related to refraction at the first surface. Because the Sun is so far away and its rays arriving at the sphere are parallel, the object distance p_a is essentially infinite and the term n_1R in the denominator can be neglected. Therefore,

$$q_a \rightarrow \frac{n_2 R p_a}{\left(n_2 - n_1\right)p_a} = \frac{n_2 R}{\left(n_2 - n_1\right)} \quad (2)$$

Now, let us consider the refraction at the second surface, for which we will use subscript *b*. We again write Equation (1) for this second refraction, but with the following considerations. For this refraction the light rays originate in the glass and exit into the surrounding material, interchanging the two indices of refraction compared to the first surface. In addition, the curvature of the glass is the opposite as for the first refraction, so the radius is negative. Let's put the radius into the equation as $-R$, so that we can just enter the absolute value of the radius for *R* into the final equation. Equation (1) can be rewritten, with these considerations:

$$q_b \rightarrow \frac{-n_1 R p_b}{\left(n_1 - n_2\right)p_b + n_2 R} = \frac{n_1 R p_b}{\left(n_2 - n_1\right)p_b - n_2 R} \quad (3)$$

All measurements for both refractions are measured from the center of the spherical surface. Therefore, $p_b = -q_a + 2R$, where the minus sign indicates that the image of the first refracting surface is on the back side of the second surface. (If we were to ignore refraction at the second surface, the Sun's rays would focus outside the sphere due to the first surface.) Evaluate this quantity using Equation (2):

$$p_b = -q_a + 2R = -\frac{n_2 R}{\left(n_1 - n_2\right)} + 2R = -\frac{n_2 R}{\left(n_2 - n_1\right)} + 2R\frac{\left(n_2 - n_1\right)}{\left(n_2 - n_1\right)}$$

$$= \frac{\left(n_2 - 2n_1\right)}{\left(n_2 - n_1\right)}R \quad (4)$$

Make this substitution into Equation (3):

$$q_b = \frac{n_1 R\left[\dfrac{(n_2 - 2n_1)}{n_2 - n_1}R\right]}{(n_2 - n_1)\left[\dfrac{(n_2 - 2n_1)}{n_2 - n_1}R\right] - n_2 R} = \frac{(2n_1 - n_2)}{2(n_2 - n_1)} \quad (5)$$

For a glass sphere in air, we can let $n_1 = 1$ and $n_2 = n$, so that

$$q_b = \boxed{\frac{(2 - n)}{2(n - 1)}R} \quad (6)$$

This distance is the position relative to the surface of the sphere at which the photocells must be placed for the Sun's rays to focus on them.

Finalize Let us look at some typical values. First, notice that if $n = 2$,

$$q_b = \frac{(2 - 2)}{2(2 - 1)}R = 0$$

The first refraction is strong enough in this case that the Sun's rays focus on the second surface. Therefore, we must have $n < 2$ for the spherical solar concentrator to work, because the rays must exit the sphere to focus on the outside to be collected.

Let us consider another case. Suppose the sphere is made from flint glass, with $n = 1.66$. Then

$$q_b == \frac{(2 - 1.66)}{2(1.66 - 1)}R = 0.258R$$

And the focus point is about a quarter of the radius from the outside surface of the sphere. Based on the location of the arc on which the solar collectors ride in Figure P35.22, can you estimate the index of refraction of the sphere material? Finally, suppose we consider the sphere of water in the fishbowl in Problem 35.21. In this case,

$$q_b == \frac{(2 - 1.33)}{2(1.33 - 1)}R = 1.015R$$

Therefore, the Sun's rays focus at about a distance of the radius away from the surface of the bowl. If the bowl is sitting on a wooden table, this focusing of the rays could occur right on the surface of the table and set it on fire!

Answer: The track must be placed a radial distance from the outer surface of $\dfrac{(2-n)}{2(n-1)}R$.

P35.34 The lens should take parallel light rays from a very distant object ($p = \infty$) and make them diverge from a virtual image at the Josh's far point, which is 25.0 cm beyond the lens, at $q = -25.0$ cm.

(a) $$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} - \frac{1}{0.250\text{ m}} = \boxed{-4.00\text{ diopters}}$$

(b) The power is negative: a $\boxed{\text{diverging lens}}$.

P35.36 $f_o = 20.0$ m, $f_e = 0.025\,0$ m

(a) From Equation 35.27, The angular magnification produced by this telescope is

$$m = -\frac{f_o}{f_e} = \boxed{-800}$$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

P35.37 Using Equation 35.26,

$$M \approx -\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right) = -\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right) = \boxed{-575}$$

P35.42 (a) The mirror-and-lens equation, $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$, gives

$$q = \frac{1}{1/f - 1/p} = \frac{1}{(p-f)/fp} = \frac{fp}{p-f}$$

Then,

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{f}{p-f}$$

gives $h' = \dfrac{fh}{f-p}$

(b) For $p \gg f$, $f - p \approx -p$. Then, $h' = \boxed{-\dfrac{hf}{p}}$

(c) Suppose the telescope observes the space station at the zenith:

$$h' = -\frac{hf}{p} = -\frac{(108.6\text{ m})(4.00\text{ m})}{407\times 10^3\text{ m}} = \boxed{-1.07\text{ mm}}$$

P35.49 Use the lens makers' equation, Equation 35.17, and the conventions of Table 35.2. The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{1,1}} - \frac{1}{R_{1,2}}\right) = (n_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = \frac{1-n_1}{R}$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_{2,1}} - \frac{1}{R_{2,2}}\right) = (n_2 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = +\frac{2(n_2-1)}{R}$$

Let an object be placed at any distance p_1 large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{1-n_1}{R} - \frac{1}{p_1}$$

This virtual ($q_1 < 0$) image (to the left of lens 1) is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{2n_2-2}{R} - \frac{1}{p_2} = \frac{2n_2-2}{R} + \frac{1}{q_1} = \frac{2n_2-2}{R} + \frac{1-n_1}{R} - \frac{1}{p_1}$$

$$= \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}$$

Then $\dfrac{1}{p_1} + \dfrac{1}{q_2} = \dfrac{2n_2 - n_1 - 1}{R}$ so the doublet behaves like a single lens with $\dfrac{1}{f} = \dfrac{2n_2 - n_1 - 1}{R}$.