P36.3 The location of the bright fringe of order m (measured from the position of the central maximum) is

$$d\sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots.$$

For first bright fringe to the side, $m = 1$. Thus, the wavelength of the laser light must be

$$\lambda = d\sin\theta = (0.200\times10^{-3}\,\text{m})\sin 0.181°$$
$$= 6.32\times10^{-7}\ \text{m} = \boxed{632\ \text{nm}}$$

P36.4 (a) For a bright fringe of order m, the path difference is $\delta = m\lambda$, where $m = 0, 1, 2, \ldots$ At the location of the third order bright fringe,

$$\delta = m\lambda = 3\left(589\times10^{-9}\ \text{m}\right) = 1.77\times10^{-6}\ \text{m} = \boxed{1.77\ \mu\text{m}}$$

(b) For a dark fringe, the path difference is $\delta = \left(m+\frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, \ldots$ At the third dark fringe, $m = 2$ and

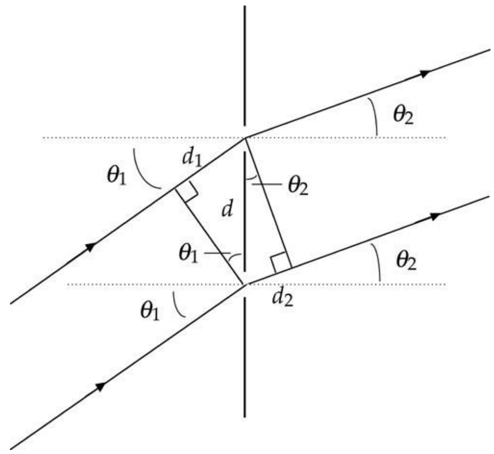
$$\delta = \left(2+\frac{1}{2}\right)\lambda = \frac{5}{2}(589\ \text{nm}) = 1.47\times10^{3}\ \text{nm} = \boxed{1.47\ \mu\text{m}}$$

P36.9 From the diagram, the path difference between rays 1 and 2 is

$$\delta = d_1 - d_2 = d\sin\theta_1 - d\sin\theta_2$$

For constructive interference, this path difference must be equal to an integral number of wavelengths:

$$d\sin\theta_1 - d\sin\theta_2 = m\lambda$$
$$\sin\theta_1 - \sin\theta_2 = \frac{m\lambda}{d} \quad\rightarrow\quad \theta_2 = \sin^{-1}\left(\sin\theta_1 - \frac{m\lambda}{d}\right)$$



ANS. FIG. P36.9

P36.13 (a) The path difference $\delta = d\sin\theta$, and when $L >> y$:

$$\delta = \frac{yd}{L} = \frac{(1.80 \times 10^{-2}\ \text{m})(1.50 \times 10^{-4}\ \text{m})}{1.40\ \text{m}}$$

$$= 1.93 \times 10^{-6}\ \text{m} = \boxed{1.93\ \mu\text{m}}$$

(b) $$\frac{\delta}{\lambda} = \frac{1.93 \times 10^{-6}\ \text{m}}{6.43 \times 10^{-7}\ \text{m}} = 3.00, \quad \text{or} \quad \boxed{\delta = 3.00\lambda}$$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

P36.15 We use trigonometric identities to write

$$E_1 + E_2 = 6.00\sin(100\pi t) + 8.00\sin(100\pi t + \pi/2)$$

$$= 6.00\sin(100\pi t) + [8.00\sin(100\pi t)\cos(\pi/2) + 8.00\cos(100\pi t)\sin(\pi/2)]$$

$$E_1 + E_2 = 6.00\sin(100\pi t) + 8.00\cos(100\pi t)$$

and

$$E_R\sin(100\pi t + \phi) = E_R\sin(100\pi t)\cos\phi + E_R\cos(100\pi t)\sin\phi$$

The equation $E_1 + E_2 = E_R\sin(100\pi t + \phi)$ is satisfied if we require

$$600 = E_R^{-}\cos\phi \quad \text{and} \quad 8.00 = E_R\sin\phi$$

or $$(6.00)^2 + (8.00)^2 = E_R{}^2(\cos^2\phi + \sin^2\phi) \rightarrow \boxed{E_R = 10.0}$$

and $$\tan\phi = \sin\phi/\cos\phi = 8.00/6.00 = 1.33 \rightarrow \boxed{\phi = 53.1^\circ}$$

***P36.16** **Conceptualize** Study Figure 36.4 carefully, so that you understand the origin of the intensity difference on the screen.

Categorize The problem involves the *waves in interference* model in the special case of two-slit interference.

Analyze Although the light waves are in phase as they leave the slits, their phase difference ϕ at P dependson the path difference according to Equation 36.1: $\delta = r_2 - r_1 = d\sin\theta$. A *path* difference of λ (for constructiveinterference) corresponds to a *phase* difference of 2π rad. Therefore, a path difference of δ isthe same fraction of λ as the phase difference ϕ is of 2π:

$$\frac{d}{l} = \frac{f}{2p} \qquad (1)$$

Solve Equation (1) for ϕ and substitute from Equation 36.1:

$$f = \frac{2p}{l}d = \frac{2p}{l}d\sin q \qquad (2)$$

Use the superposition principle to combine the electric field magnitudes given in the problem statement to find an expression for the magnitude of the resultant electric field at point P:

$$E_P = E_1 + E_2 = E_0 \sin wt + E_0 \sin\left(wt + f\right) = E_0\left[\sin wt + \sin\left(wt + f\right)\right] \qquad (3)$$

We can simplify this expression by using the trigonometric identity

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \qquad (4)$$

Identifying A as $(\omega t + \phi)$ and B as ωt, use Equation (4) to rewrite Equation (3):

$$E_P = 2E_0 \sin\left[\frac{\left(wt + f\right) + wt}{2}\right]\cos\left[\frac{\left(wt + f\right) - wt}{2}\right]$$

$$= 2E_0 \cos\left(\frac{f}{2}\right)\sin\left(wt + \frac{f}{2}\right) \qquad (5)$$

The intensity of the light is proportional to the square of the electric field magnitude, so

$$I(t) \propto E_P^2 = 4E_0^2 a^2 \cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\phi}{2}\right) \qquad (6)$$

where a^2 is a constant of proportionality. The intensity seen or measured on a screen will be the time-averaged intensity, so integrate the time-dependent intensity in Equation (6) over one cycle:

$$I = \frac{1}{T}\int_0^T 4E_0^2 a^2 \cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\phi}{2}\right) dt = \frac{1}{T} 4E_0^2 a^2 \cos^2\left(\frac{\phi}{2}\right)\int_0^T \sin^2\left(\omega t + \frac{\phi}{2}\right) dt$$

$$= \frac{1}{T} 4E_0^2 a^2 \cos^2\left(\frac{\phi}{2}\right)\left(\tfrac{1}{2}\right) = I_{\max}\cos^2\left(\frac{\phi}{2}\right) \qquad (7)$$

where we have gathered all of the constants together and identified the combination of constants as $I_{\max}$. The evaluation of the integral as $\frac{1}{2}$ uses the same argument as that associated with Figure 32.5.

Finally, substitute Equation (2) into Equation (7):

$$I = I_{\max}\cos^2\left[\frac{\left(\dfrac{2\pi}{\lambda} d\sin\theta\right)}{2}\right] = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) \qquad (8)$$

Finalize Equation (8) is identical to Equation 36.9 in the text.]

P36.19 There are a total of two phase reversals caused by reflection, one at the top and one at the bottom surface of the coating.

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad \text{so} \quad t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$$

The minimum thickness of the film is therefore

$$t = \left(\frac{1}{2}\right)\frac{(500 \text{ nm})}{2(1.30)} = \boxed{96.2 \text{ nm}}$$

***P36.24** **Conceptualize** The radius of the circular oil slick will allow us to find its area. The oil acts as a thin film floating on the water surface. The optical information will allow us to find its thickness. From these measurements, we can find the volume of the oil that was spilled.

Categorize The light waves will be modeled as *waves in interference* in the special case of interference on thin films.

Analyze Because the index of refraction of the oil is between that of air and water, there will be a 180° phase shift both at the reflection of light from the oil surface and at the water surface beneath the oil. Therefore, the condition for constructive interference for light reflected from the oil is given by Equation 36.13:

$$2nt = ml \quad \rightarrow \quad t = \frac{ml}{2n} \qquad (1)$$

The oil forms a very thin disk around the location at which the oil spilled, so the volume of the oil is

$$V = At = \left(pr^2\right)\left(\frac{ml}{2n}\right) = p\frac{ml\,r^2}{2n} \qquad (2)$$

Substitute numerical values, assuming the minimum order number of $m = 1$:

$$V = p\frac{(1)(500\text{ nm})\left(4.25\times10^3\text{ m}\right)^2}{2(1.25)}\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right) = \boxed{11.3\text{ m}^3}$$

Finalize This result is larger than the limit of 10.0 m^3 indicated in the problem statement. We assumed the interference observed was represented by $m = 1$. Because the volume is proportional to m, if the interference represents a higher value of m, the volume of oil spilled is even larger. In reality, the measurement would be complicated by variations of thickness in the oil and the effects of wave motion on the slick.

Answer: 11.3 m^3

P36.36 From Figure P36.35, observe that the distance that the ray travels from the top of the transmitter to the ground is

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves (transmitter-to-ground-to-receiver and transmitter-to-receiver) is

$$\delta = 2x + \frac{\lambda}{2} - d$$

For constructive interference,

$$2x+\frac{\lambda}{2}-d=m\lambda \;\rightarrow\; \lambda=\frac{2x-d}{\left(m-\frac{1}{2}\right)}$$

and for destructive interference

$$2x+\frac{\lambda}{2}-d=\left(m+\frac{1}{2}\right)\lambda \;\rightarrow\; \lambda=\frac{2x-d}{m}$$

(a) The longest wavelength that interferes constructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{\left(1-\frac{1}{2}\right)}=4x-2d=\frac{4\sqrt{4h^2+d^2}}{2}-2d=\boxed{2\sqrt{4h^2+d^2}-2d}$$

(b) The longest wavelength that interferes destructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{1}=\boxed{\sqrt{4h^2+d^2}-d}$$

***P36.43** **Conceptualize** The important information in the problem statement is that the reflections of the two halves of the laser beam from the flat and the pit must undergo destructive interference. This will determine the depth of the pit.

Categorize The two halves of the reflected laser light are modeled as *waves in interference.*

Analyze Both halves of the beam reflect from the protective coating, so they either both undergo a 180° phase shift or neither one does. The wavelength difference of the two reflected portions of the laser beam, therefore, will be determined only by the extra distance that one half of the beam has to travel to reach the flat and come back downward. If the depth of the pit is d, in order to have destructive interference, we must have

$$\left(m+\tfrac{1}{2}\right)\lambda_n=2d \quad\rightarrow\quad d=\tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_n=\tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n} \qquad (1)$$

where we have incorporated Equation 34.6. For the lowest integer values of $m$we substitute numerical values:

$$d_0 = \tfrac{1}{2}\left(0+\tfrac{1}{2}\right)\left(\frac{200\text{ nm}}{1.78}\right) = 28.1\text{ nm}$$

$$d_1 = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\left(\frac{200\text{ nm}}{1.78}\right) = 84.3\text{ nm}$$

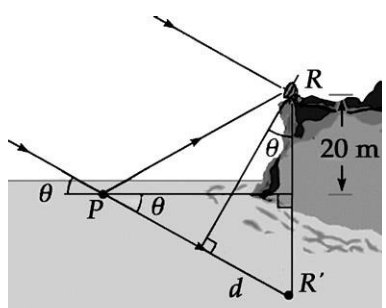
$$d_2 = \tfrac{1}{2}\left(2+\tfrac{1}{2}\right)\left(\frac{200\text{ nm}}{1.78}\right) = \boxed{140\text{ nm}}$$

The first value of m to meet the manufacturing limitation of 0.1 μm is $m = 2$.

Finalize If the manufacturing limitation of 0.1 μm is reduced with future technology, then perhaps values of $m = 0$ or $m = 1$ could be used.]

Answer: 140 nm

P36.45 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P. The distance from P to R is the same as from P to R', where R' is the mirror image of the telescope. Therefore, the path difference is d.



ANS. FIG. P36.45

Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad \textbf{[1]}$$

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

So the path difference is

$$d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$$

The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{60.0\times10^6\text{ Hz}} = 5.00\text{ m}$$

Substituting for d and λ in equation [1],

$$(40.0\text{ m})\sin\theta = \frac{5.00\text{ m}}{2}$$

Solving for the angle θ,

$$\theta = \sin^{-1}\left(\frac{5.00\text{ m}}{80.0\text{ m}}\right) = \boxed{3.58^\circ}$$