P37.3 In a single slit diffraction pattern, with the slit having width a, the dark fringe of order m occurs at angle θ_m, where $\sin\theta_m = m(\lambda/a)$ and $m = \pm 1,\ \pm 2,\ \pm 3, \ldots$. The location, on a screen located distance L from the slit, of the dark fringe of order m (measured from $y = 0$ at the center of the central maximum) is

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda\left(\frac{L}{a}\right)$$

(a) The central maximum extends from the $m = +1$ dark fringe on one side to the $m = -1$ dark fringe on the other side, so the width of this central maximum is

$$\begin{aligned}\text{Central max. width} &= (y_{\text{dark}})_{m=1} - (y_{\text{dark}})_{m=-1} \\ &= (1)\left(\frac{\lambda L}{a}\right) - (-1)\left(\frac{\lambda L}{a}\right) = \frac{2\lambda L}{a}\end{aligned}$$

Therefore,

$$\begin{aligned}L &= \frac{a(\text{Central max. width})}{2\lambda} \\ &= \frac{(0.200\times 10^{-3}\ \text{m})(8.10\times 10^{-3}\ \text{m})}{2(5.40\times 10^{-7}\ \text{m})} = \boxed{1.50\ \text{m}}\end{aligned}$$

(b) The first order bright fringe extends from the $m = 1$ dark fringe to the $m = 2$ dark fringe, or

$$\begin{aligned}(\Delta y_{\text{bright}})_1 &= (y_{\text{dark}})_{m=2} - (y_{\text{dark}})_{m=1} = 2\left(\frac{\lambda L}{a}\right) - 1\left(\frac{\lambda L}{a}\right) = \frac{\lambda L}{a} \\ &= \frac{(5.40\times 10^{-7}\ \text{m})(1.50\ \text{m})}{0.200\times 10^{-3}\ \text{m}} \\ &= 4.05\times 10^{-3}\ \text{m} = \boxed{4.05\ \text{mm}}\end{aligned}$$

Note that the width of the first order bright fringe is exactly one half the width of the central maximum.

P37.7 First we find where we are. The angle to the side is small so

$$\sin\theta \approx \tan\theta = \frac{y}{L} = \frac{4.10 \times 10^{-3}\text{ m}}{1.20\text{ m}} = 3.417 \times 10^{-3}$$

The parameter controlling the intensity is

$$\frac{\pi a \sin\theta}{\lambda} = \frac{\pi\left(4.00 \times 10^{-4}\text{ m}\right)\left(3.417 \times 10^{-3}\right)}{546.1 \times 10^{-9}\text{ m}} = 7.862\text{ rad}$$

This is between 2ρ and 3π, so the point analyzed is off in the second side fringe. The fractional intensity is

$$\frac{I}{I_{\max}} = \left[\frac{\sin\left(\pi a \sin\theta/\lambda\right)}{\pi a \sin\theta/\lambda}\right]^2 = \left[\frac{\sin(7.862\text{ rad})}{7.862\text{ rad}}\right]^2 = \boxed{1.62 \times 10^{-2}}$$

P37.9 Using Rayleigh's criterion, $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{y}{L}$. Therefore,

$$y = 1.22\left(\frac{\lambda}{D}\right)L = 1.22\left(\frac{500 \times 10^{-9}\text{ m}}{58.0 \times 10^{-2}\text{ m}}\right)\left(270 \times 10^{3}\text{ m}\right)$$

$$= \boxed{0.284\text{ m}}$$

P37.11 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. We are given

$$L = 250 \times 10^{3}\text{ m},\ \lambda = 5.00 \times 10^{-7}\text{ m, and } d = 5.00 \times 10^{-3}\text{ m}$$

The smallest object the astronauts can resolve is given by Rayleigh's criterion, $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{y}{L}$. Therefore,

$$y = 1.22\frac{\lambda}{D}L = 1.22\left(\frac{5.00 \times 10^{-7}\text{ m}}{5.00 \times 10^{-3}\text{ m}}\right)\left(250 \times 10^{3}\text{ m}\right) = \boxed{30.5\text{ m}}$$

P37.13 The limit of resolution in air is

$$\theta_{\min}\Big|_{\text{air}} = 1.22\frac{\lambda}{D} = 0.60\ \mu\text{rad}$$

In oil, the limiting angle of resolution will be

$$\theta_{\min}\Big|_{\text{oil}} = 1.22\frac{\lambda_{\text{oil}}}{D} = 1.22\frac{(\lambda/n_{\text{oil}})}{D} = \frac{1}{n_{\text{oil}}}\left(1.22\frac{\lambda}{D}\right)$$

or $$\theta_{\min}\Big|_{\text{oil}} = \frac{\theta_{\min}\big|_{\text{air}}}{n_{\text{oil}}} = \frac{0.60\ \mu\text{rad}}{1.5} = \boxed{0.40\ \mu\text{rad}}$$

P37.15 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. According to this criterion, two dots separated center-to-center by 2.00 mm would overlap when

$$\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $$L = \frac{dD}{1.22\lambda} = \frac{(2.00\times 10^{-3}\ \text{m})(5.00\times 10^{-3}\ \text{m})}{1.22(500\times 10^{-9}\ \text{m})} = \boxed{16.4\ \text{m}}.$$

P37.19 The grating spacing is

$$d = \frac{1.00\times 10^{-3}\ \text{m}}{250} = 4.00\times 10^{-6}\ \text{m} = 4\ 000\ \text{nm}$$

Solving for m in Equation 38.7 gives

$$d\sin\theta = m\lambda \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda}$$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$$

or $\boxed{\text{5 orders is the maximum}}$.

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$$

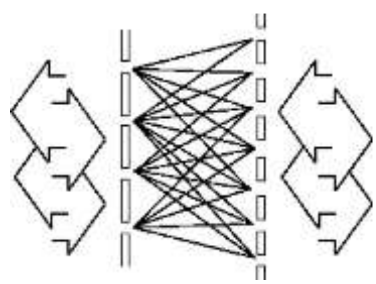
or $\boxed{\text{10 orders in the short-wavelength region}}$.

P37.22 (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first-order maxima are separated according to

$$d\sin\theta = (1)\lambda \qquad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\ \text{m}}{1.2\times10^{-3}\ \text{m}}$$

$$\theta = \sin^{-1}(0.000\ 527) = 0.000\ 527\ \text{rad}$$

$$y = L\tan\theta = (1.40\ \text{m})(0.000\ 527) = \boxed{0.738\ \text{mm}}$$



ANS. FIG. P37.22

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$$d\sin\theta = (1)\lambda \quad\rightarrow\quad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\ \text{m}}{0.738\times10^{-3}\ \text{m}} = 0.000\ 857$$

$$y = L\tan\theta = (1.40 \text{ m})(0.000\,857) = 1.20 \text{ mm}$$

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left.

P37.25 (a) By Bragg's law, $2d\sin\theta = m\lambda$, and $m = 2$:

$$\lambda = 2d\sin\theta = 2(0.250 \text{ nm})\sin 12.6° = \boxed{0.109 \text{ nm}}$$

(b) We obtain the number of orders from

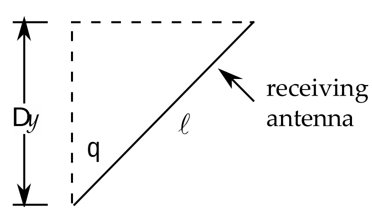
$$\frac{m\lambda}{2d} = \sin\theta \le 1 \;\rightarrow\; m \le \frac{2d}{\lambda} = \frac{2(0.250 \text{ nm})}{0.109 \text{ nm}} = 4.59$$

The order-number must be an integer, so the largest value m can have is 4: $\boxed{\text{four}}$ orders can be observed.

P37.27 $P = \dfrac{(\Delta V)^2}{R}$ or $P \propto (\Delta V)^2$

$$\Delta V = (-)E_y \cdot \Delta y = E_y \cdot \ell\cos\theta$$

$$\Delta V \propto \cos\theta \quad \text{so} \quad P \propto \cos^2\theta$$



ANS. FIG. P37.27

(a) $\theta = 15.0°$:

$$P = P_{\max}\cos^2(15.0°) = 0.933P_{\max} = \boxed{93.3\%}$$

(b) $\theta = 45.0°$: $P = P_{\max}\cos^2(45.0°) = 0.500P_{\max} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $P = P_{\max}\cos^2(90.0°) = \boxed{0.00\%}$

P37.30 For the polarizing angle,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n}{1} = n$$

and $$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{n}$$

Thus, $$\tan\theta_p = \frac{1}{\sin\theta_c}:$$

$$\boxed{\theta_p = \tan^{-1}\left(\frac{1}{\sin\theta_c}\right) \quad \text{or} \quad \theta_p = \tan^{-1}(\csc\theta_c) \quad \text{or} \quad \theta_p = \cot^{-1}(\sin\theta_c)}$$

P37.32 (a) Let I_0 represent the intensity of unpolarized light incident on the first polarizer. The intensity of unpolarized light passing through a polarizing filter is reduced by 1/2, so the first filter lets through 1/2 of the incident intensity. Of the light reaching them, the second filter passes $\cos^2 45° = 1/2$ and the third filter also $\cos^2 45° = 1/2$. The transmitted intensity is then

$$I_0\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.125I_0$$

The reduction in intensity is by a factor of $1.00 - 0.125 = \boxed{0.875}$ of the incident intensity.

(b) By the same logic as in part (a) we have transmitted

$$I_0\left(\frac{1}{2}\right)\left(\cos^2 30.0°\right)\left(\cos^2 30.0°\right)\left(\cos^2 30.0°\right)=\left(\frac{I_0}{2}\right)\left(\cos^2 30.0°\right)^3$$

$$=0.211I_0$$

Then the fraction absorbed is 1.00 – 0.211 = $\boxed{0.789}$.

(c) Yet again we compute transmission

$$I_0\left(\frac{1}{2}\right)\left(\cos^2 15.0°\right)^6=0.330I_0$$

And the fraction absorbed is 1.00 – 0.330 = $\boxed{0.670}$.

(d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

P37.37 (a) We first determine the wavelength of 1.40-GHz radio waves from $\lambda=\frac{v}{f}$:

$$\lambda=\frac{3.00\times10^8\text{ m/s}}{1.40\times10^9\text{ s}^{-1}}=0.214\text{ m}$$

Applying Rayleigh's criterion, $\theta_{\min}=1.22\frac{\lambda}{D}$, we obtain

$$\theta_{\min}=1.22\left(\frac{0.214\text{ m}}{3.60\times10^4\text{ m}}\right)=\boxed{7.26\ \mu\text{rad}}$$

$$\theta_{\min}=(7.26\ \mu\text{rad})\left(\frac{180\times60\times60\text{ s}}{\pi}\right)=\boxed{1.50\text{ arc seconds}}$$

(b) To determine the separation between the clouds, we use $\theta_{\min}=\frac{d}{L}$:

$$d=\theta_{\min}L=\left(7.26\times10^{-6}\text{ rad}\right)\left(26\,000\text{ ly}\right)=\boxed{0.189\text{ ly}}$$

(c) It is not true for humans, but we assume the hawk's visual acuity is limited only by Rayleigh's criterion, $\theta_{\min} = 1.22\frac{\lambda}{D}$. Substituting numerical values,

$$\theta_{\min} = 1.22\left(\frac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = 50.8\ \mu\text{rad} = \boxed{10.5\text{ seconds of arc}}$$

(d) Following the same procedure as in part (b), we have

$$d = \theta_{\min}L = \left(50.8\times10^{-6}\text{ rad}\right)(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$$

P37.43 (a) We require

$$\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D/2}{L}.$$

Then, $\boxed{D^2 = 2.44\lambda L}$.

(b) $D = \sqrt{2.44\left(500\times10^{-9}\text{ m}\right)(0.150\text{ m})} = 4.28\times10^{-4}\text{ m} = \boxed{428\ \mu\text{m}}$

P37.44 (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu$m apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$$2\left(0.25\times10^{-6}\text{ m}\right)\sin 90° = \lambda = 500\text{ nm}$$

This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $2\left(0.25\times10^{-6}\text{ m}\right)\sin 60° = \lambda = 433\text{ nm}$. This is violet.

(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say 0.32 μm) they could reflect red with constructive interference.

P37.50 (a) The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = \theta_{\min} = 1.22\frac{\lambda}{D}$$

$$y = \left(2\times 10^{7}\text{ m}\right)(1.22)\left(\frac{6\times 10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. A resolution of about 3 cm would make it difficult to read a license plate.

(b) No. The resolution is too large. It cannot count coins spilled on a sidewalk, much less read the dates on them.

Considering atmospheric image distortion caused by variations in air density and temperature, the distance between barely resolvable objects is more like, assuming a limiting angle of one second of arc,

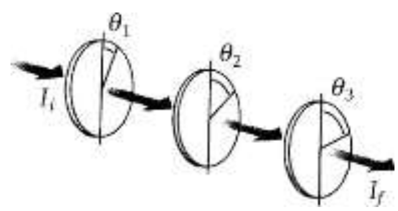
$$\left(2\times10^{7}\text{ m}\right)(1\text{ s})\left(\frac{1^\circ}{3\,600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180^\circ}\right)=97\text{ cm}\approx1\text{ m}$$

P37.52 For incident unpolarized light of intensity I_{max}, the average value of the cosine-squared function is one-half, so the intensity after transmission by the first disk is $I=\frac{1}{2}I_{max}$.

After transmitting 2nd disk: $I=\frac{1}{2}I_{max}\cos^2\theta$

After transmitting 3rd disk: $I=\frac{1}{2}I_{max}\cos^2\theta\cos^2(90^\circ-\theta)$

where the angle between the first and second disk is $\theta=\omega t$.



ANS. FIG. P37.52

Using trigonometric identities $\cos^2\theta=\frac{1}{2}(1+\cos2\theta)$

and $\cos^2(90^\circ-\theta)=\sin^2\theta=\frac{1}{2}(1-\cos2\theta)$,

we have $I=\frac{1}{2}I_{max}\left[\frac{(1+\cos2\theta)}{2}\right]\left[\frac{(1-\cos2\theta)}{2}\right]$

$$I=\frac{1}{8}I_{max}\left(1-\cos^2 2\theta\right)=\frac{1}{8}I_{max}\left(\frac{1}{2}\right)(1-\cos4\theta)$$

Since $\theta=\omega t$, the intensity of the emerging beam is given by

$$\boxed{I=\frac{1}{16}I_{max}\left(1-\cos\,4\omega t\right)}$$