P38.3 (a) The length of the meter stick measured by the observer moving at speed $v = 0.900\,c$ relative to the meter stick is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (1.00\text{ m})\sqrt{1-(0.900)^2} = \boxed{0.436\text{ m}}$$

(b) If the observer moves relative to Earth in the direction opposite the motion of the meter stick relative to Earth, the velocity of the observer relative to the meter stick is greater than that in part (a). The measured length of the meter stick will be $\boxed{\text{less than }0.436\text{ m}}$ under these conditions, but so small it is unobservable.

P38.4 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$.

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{L_P}{v} = \frac{4.60\text{ km}}{0.990c} = \left[\frac{4.60\times 10^3\text{ m}}{0.990\left(3.00\times 10^8\text{ m/s}\right)}\right]$$
$$= 1.55\times 10^{-5}\text{ s} = 15.5\ \mu\text{s}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}(15.5\ \mu\text{s}) = \boxed{2.18\ \mu\text{s}}$$

(b) In the muon's frame, the Earth is approaching the muon at speed $v = 0.990c$. During the time interval the muon exists, the Earth travels the distance

$$d = v\Delta t_p = v\frac{\Delta t}{\gamma} = v\frac{L_P}{\gamma v} = \frac{L_P}{\gamma}$$
$$= \left(4.60\times 10^3\text{ m}\right)\sqrt{1-(0.990)^2} = \boxed{649\text{ m}}$$

P38.7 From the definition of γ,

$$\gamma = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}} = 1.010\ 0$$

we solve for the speed:

$$v = c\sqrt{1-\left(\frac{1}{\gamma}\right)^2} = c\sqrt{1-\left(\frac{1}{1.010\ 0}\right)^2} = \boxed{0.140c}$$

***P38.8** **Conceptualize** The driver is claiming an Earth-based version of the Doppler shift for light, which causes light from galaxies receding from us to experience a redshift. Because the driver is moving toward the light source, the wavelength should be shifted toward the blue end of the spectrum.

Categorize We use our understanding of the Doppler effect for light.

Analyze Solve Equation 38.10 for the relative speed between the driver and the traffic light when they are moving toward each other:

$$f' = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}f \quad \rightarrow \quad v = \frac{\left(f'\right)^2 - f^2}{\left(f'\right)^2 + f^2}c \qquad (1)$$

Express Equation (1) in terms of wavelengths:

$$v = \frac{\left(\frac{c}{\lambda'}\right)^2 - \left(\frac{c}{\lambda}\right)^2}{\left(\frac{c}{\lambda'}\right)^2 + \left(\frac{c}{\lambda}\right)^2}c = \frac{\lambda^2 - \left(\lambda'\right)^2}{\lambda^2 + \left(\lambda'\right)^2}c \qquad (2)$$

Substitute numerical values:

$$v = \frac{\left(650\text{ nm}\right)^2 - \left(520\text{ nm}\right)^2}{\left(650\text{ nm}\right)^2 + \left(520\text{ nm}\right)^2}c = 0.220c$$

A speed of 22% of the speed of light is well over the speed limit of any Earth-based road. The driver's own testimony shows him blatantly violating any Earth-based speed limit; look for another defense.

Finalize The speed found in the problem is equivalent to 6.59×10^7 m/s, or 1.47×10^8 mi/h. Such speeds are not possible with automobiles or other Earth-based vehicles, so it is clear that the Doppler shift of light is not a phenomenon with which we have to deal on an everyday basis.]

Answer: The driver's own testimony shows him blatantly violating any Earth-based speed limit; look for another defense.

P38.10 The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$

and $L = v\Delta t$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for v,

$$v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2$$

giving $$\boxed{v = \frac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$$

P38.13 We find Cooper's speed from Newton's second law:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving,

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m} + 0.160\times10^{6}\ \text{m}\right)}\right]^{1/2}$$
$$= 7.82\times10^3 = 7.82\ \text{km/s}$$

Then the time period of one orbit is

$$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6\ \text{m})}{7.82\times10^3\ \text{m/s}} = 5.25\times10^3\ \text{s}$$

(a) The time difference for 22 orbits is

$$\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$$

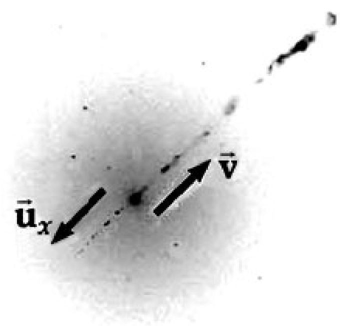
$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2$$
$$\times 22\left(5.25\times10^3\ \text{s}\right) = \boxed{39.2\ \mu s}$$

(b) For each one orbit Cooper aged less by

$$\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$$

The press report is $\boxed{\text{accurate to one digit}}$.

P38.21 Take the galaxy as the unmoving frame. Arbitrarily define the jet moving upward to be the object, and the jet moving downward to be the "moving" frame:



ANS. FIG. P38.21

$u'_x =$ velocity of other jet in frame of jet

$u_x =$ velocity of other jet in frame of galaxy center

$= 0.750c$

$v =$ speed of galaxy center in frame of jet $= -0.750c$

From Equation 38.16, the speed of the upward-moving jet as measured from the downward-moving jet is

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.750c - (-0.750c)}{1 - (0.750c)(-0.750c)/c^2} = \frac{1.50c}{1 + 0.750^2}$$

$$= \boxed{0.960c}$$

P38.27 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$,

or $$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50 \times 10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}} \times (0.893c)$$

or $$\frac{(1.67 \times 10^{-27}\ \text{kg})u_2}{\sqrt{1-(u_2/c)^2}} = (4.960 \times 10^{-28}\ \text{kg})c$$

Proceeding to solve, we find

$$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2 = 1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2}=1 \qquad \text{and} \qquad u_2 = \boxed{0.285c}$$

P38.29 We use the equation $\Delta E = (\gamma_1 - \gamma_2)mc^2$. For an electron, $mc^2 = 0.511$ MeV.

(a) $$\Delta E = \left(\sqrt{\frac{1}{(1-0.810)}} - \sqrt{\frac{1}{(1-0.250)}}\right)mc^2 = \boxed{0.582\text{ MeV}}$$

(b) $$\Delta E = \left(\sqrt{\frac{1}{1-(0.990)^2}} - \sqrt{\frac{1}{1-0.810}}\right)mc^2 = \boxed{2.45\text{ MeV}}$$

P38.30 The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2$. The classical equation is $K_c = \frac{1}{2}mu^2$. Their ratio is

$$\frac{K_r}{K_c} = \frac{\left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2}{\frac{1}{2}mu^2} = \frac{2\left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)}{u^2/c^2}$$

$$= 2\left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)\frac{1}{u^2/c^2}$$

$$\frac{K_r}{K_c} = 2\left(\frac{1}{\sqrt{1-(0.100)^2}}-1\right)\frac{1}{(0.100)^2} = 1.007\ 56$$

For still smaller speeds the agreement will be still better.

P38.31 (a) To find the speed of the protons with $E = \gamma mc^2 = 400mc^2$, we write

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} \quad \rightarrow \quad u = c\sqrt{1-\frac{1}{\gamma^2}}$$

So, $$u = c\sqrt{1-\frac{1}{(400)^2}} = \boxed{0.999\,997c}$$

(b) From Example 38.9, for a proton, $mc^2 = 938$ MeV. Then

$$K = (\gamma - 1)mc^2 = 399(938 \text{ MeV}) = \boxed{3.74\times 10^5 \text{ MeV}}$$

***P38.32 Conceptualize** Wouldn't it be wonderful if such a reactor existed? You would simply feed the matter and antimatter into the machine and they would annihilate, releasing their *entire* rest energy. In reality, of course, we don't have a supply of a large amount of antimatter. While antimatter is produced in particle accelerators and stored for future experiments, the total amount of antimatter produced in this way would not boil a cup of tea if annihilated completely with matter. Producing enough antimatter for the proposal in this problem would be horrendously cost-prohibitive, but let's carry on anyway!

Categorize The matter and antimatter would be modeled as an *isolated system* for *energy,* in which the rest energy transforms entirely to other forms, all of which can theoretically be used to supply the world's energy needs.

Analyze (a) Solve Equation 38.24 for the mass needed to transform the rest energy into other forms of energy:

$$E_R = mc^2 \quad \rightarrow \quad m = \frac{E_R}{c^2} \qquad (1)$$

Substitute numerical values for the transformed rest energy to be equal to the world's energy requirement for one year:

$$m = \frac{4.0 \times 10^{20}\text{ J}}{\left(3.00 \times 10^{8}\text{ m/ s}\right)^{2}} = 4.4 \times 10^{3}\text{ kg}$$

This is the total mass to be transformed. Half of this mass is matter and half is antimatter, so

$$m_{\text{matter}} = \boxed{2.2 \times 10^{3}\text{ kg}}$$

$$m_{\text{antimatter}} = \boxed{2.2 \times 10^{3}\text{ kg}}$$

(b) Now, let’s address the 5.0-yr supply. The amount of mass of either matter or antimatter that will have to be stored is

$$m_{5.0\text{ yr}} = \left(5.0\text{ yr}\right)\left(2.2 \times 10^{3}\text{ kg/ yr}\right) = 1.1 \times 10^{4}\text{ kg}$$

Use Equation 1.1 to find the volume in which this mass must be stored:

$$\rho = \frac{m}{V} \quad \rightarrow \quad V = \frac{m}{\rho} = \frac{1.1 \times 10^{4}\text{ kg}}{2\,700\text{ kg/ m}^{3}} = \boxed{4.1\text{ m}^{3}}$$

Therefore, each of the matter and antimatter could be stored in a cubic container about 1.6 m on a side.

Finalize How realistic is this proposal? Not realistic at all, at least with today’s technology and scientific understanding. The concept of generating energy with no waste is optimistic. Even if we imagine that waste energy in the form of internal energy could be used to warm houses and commercial buildings, how do we transport that energy to the site? How do we feed the matter and antimatter in at a slow enough rate so that the reactor does not explode? How can we store antimatter on a world made of matter?]

Answers: (a) 2.2×10^{3} kg for each of matter and antimatter (b) 4.1 m^{3} for each of matter and antimatter

P38.33 Given $E = 2mc^2$, where $mc^2 = 938$ MeV from Example 38.9. We use Equation 38.27:

$$E^2 = p^2c^2 + (mc^2)^2$$

$$(2mc^2)^2 = p^2c^2 + (mc^2)^2$$

$$4(mc^2)^2 = p^2c^2 + (mc^2)^2 \quad \rightarrow \quad p^2c^2 = 3(mc^2)^2$$

Solving for the momentum then gives

$$p = \sqrt{3}\frac{(mc^2)}{c} = \sqrt{3}\frac{(938 \text{ MeV})}{c} = \boxed{1.62\times 10^3 \text{ MeV}/c}$$

P38.36 We are told to start from $E = \gamma mc^2$ and $p = \gamma mu$. Squaring both equations gives

$$E^2 = (\gamma mc^2)^2 \qquad \text{and} \qquad p^2 = (\gamma mu)^2$$

We choose to multiply the second equation by c^2 and subtract it from the first:

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2c^2$$

We factor to obtain

$$E^2 - p^2c^2 = \gamma^2\left[(mc^2)(mc^2) - (mc^2)(mu^2)\right]$$

Extracting the (mc^2) factors gives

$$E^2 - p^2c^2 = \gamma^2(mc^2)^2\left(1 - \frac{u^2}{c^2}\right)$$

We substitute the definition of γ:

$$E^2 - p^2c^2 = \left(1 - \frac{u^2}{c^2}\right)^{-1}(mc^2)^2\left(1 - \frac{u^2}{c^2}\right)$$

The γ^2 factors divide out, leaving

$$E^2 - p^2c^2 = (mc^2)^2$$

P38.41 (a) For the satellite, Newton's second law gives

$$\sum F = ma: \quad \frac{GM_E m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

which gives

$$GM_E T^2 = 4\pi^2 r^3$$

Solving for the orbital radius,

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

$$r = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(43\,080\ \text{s}\right)^2}{4\pi^2}\right]^{1/3}$$

$$= \boxed{2.66\times10^7\ \text{m}}$$

(b) $$v = \frac{2\pi r}{T} = \frac{2\pi\left(2.66\times10^7\ \text{m}\right)}{43\,080\ \text{s}} = \boxed{3.87\times10^3\ \text{m/s}}$$

(c) From the relationship of frequency and period:

$$f = \frac{1}{T} \quad \rightarrow \quad df = -\frac{dT}{T^2} = -f\left(\frac{dT}{T}\right) \quad \rightarrow \quad \frac{df}{f} = -\frac{dT}{T}$$

We see the fractional decrease in frequency is equal in magnitude to the fractional change in period.

The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\frac{df}{f} = -\frac{dT}{T} = -\frac{\gamma\Delta t_P - \Delta t_P}{\Delta t_P} = -(\gamma - 1)$$

$$\frac{df}{f} = -\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = 1 - \frac{1}{\sqrt{1-(v/c)^2}}$$

$$\approx 1 - \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right] = -\frac{1}{2}\left(\frac{v}{c}\right)^2$$

$$= -\frac{1}{2}\left[\left(\frac{3.87 \times 10^3 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2\right] = \boxed{-8.34 \times 10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use

$$U_g = -\frac{GM_E m}{r}$$

The change in gravitational potential energy is

$$\Delta U_g = -\left(6.67 \times 10^{-11} \ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)$$
$$\times \left(5.98 \times 10^{24} \text{ kg}\right) m \left[\frac{1}{2.66 \times 10^7 \text{ m}} - \frac{1}{6.37 \times 10^6 \text{ m}}\right]$$
$$= \left(4.76 \times 10^7 \text{ J/kg}\right) m$$

Then

$$\frac{\Delta f}{f} = \frac{\Delta U_g}{mc^2} = \frac{\left(4.76 \times 10^7 \text{ J/kg}\right) \cancel{m}}{\cancel{m}\left(3.00 \times 10^8 \text{ m/s}\right)^2} = \boxed{+5.29 \times 10^{-10}}$$

(e) $-8.34 \times 10^{-11} + 5.29 \times 10^{-10} = \boxed{+4.46 \times 10^{-10}}$

P38.43 (a) Observers on Earth measure the distance to Andromeda to be

$$d = 2.00 \times 10^6 \text{ ly} = \left(2.00 \times 10^6 \text{ ly}\right)c$$

The time for the trip, in Earth's frame of reference, is

$$\Delta t = \gamma \Delta t_p = \frac{30.0 \text{ yr}}{\sqrt{1-(v/c)^2}}$$

The required speed is then

$$v = \frac{d}{\Delta t} = \frac{\left(2.00 \times 10^6 \text{ ly}\right)c}{(30.0 \text{ yr})/\sqrt{1-(v/c)^2}}$$

which gives, suppressing units,

$$\left(1.50 \times 10^{-5}\right)(v/c) = \sqrt{1-(v/c)^2}$$

Squaring both sides of this equation and solving for v/c yields

$$\frac{v}{c} = \frac{1}{\sqrt{1+2.25 \times 10^{-10}}}$$

Then, the approximation $\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2}$ gives

$$\frac{v}{c} = 1 - \frac{2.25 \times 10^{-10}}{2} = \boxed{1 - 1.12 \times 10^{-10}}$$

P38.49 (a) The speed of light in water is $c/1.33$, so the electron's speed is $1.10c/1.333$. Then

$$\gamma = \frac{1}{\sqrt{1-(1.10/1.333)^2}} = 1.770$$

and the total energy is

$$E = \gamma mc^2 = 1.770(0.511 \text{ MeV}) = \boxed{0.905 \text{ MeV}}$$

(b) The electron's kinetic energy is

$$K = E - mc^2 = 0.905 \text{ MeV} - 0.511 \text{ MeV} = \boxed{0.394 \text{ MeV}}$$

(c) The electron's momentum is found from

$$pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{\gamma^2 - 1}\ \ mc^2$$
$$= \sqrt{\gamma^2 - 1}(0.511 \text{ MeV}) = 0.747 \text{ MeV}$$

and

$$p = \boxed{0.747 \frac{\text{MeV}}{c}} = \frac{0.747 \times 10^6 \left(1.602 \times 10^{-19} \text{ J}\right)}{3.00 \times 10^8 \text{ m/s}}$$
$$= \boxed{3.99 \times 10^{-22} \text{ kg} \cdot \text{m/s}}$$

(d) From Figure 16.26, the angle between the particle (source of waves) and the shock wave is

$$\sin\theta = v/v_S$$

where v is the wave speed, which is the speed of light in water, and v_S is the source speed. Then

$$\sin\theta = v/v_S = 1/1.10 \quad \rightarrow \quad \theta = \boxed{65.4°}$$

P38.51 (a) Assuming the Sun-mass system is isolated, the energy (work) required to remove a mass m from the Sun's surface to infinity is equal to the change in potential energy of the system. If the work equals the rest energy mc^2, then

$$W = \Delta E = \Delta K + \Delta U = 0 + \left(U_f - U_i\right)$$
$$mc^2 = 0 - \left(-\frac{GM_s m}{R_g}\right)$$
$$mc^2 = \frac{GM_s m}{R_g} \quad \rightarrow \quad R_g = \frac{GM_s}{c^2}$$

(b) $$R_g = \frac{GM_s}{c^2} = \frac{\left(6.67 \times 10^{-11} \text{ N.m}^2 / \text{kg}^2\right)\left(1.99 \times 10^{30} \text{ kg}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)^2}$$

$$R_g = 1.47 \times 10^3 \text{ m} = \boxed{1.47 \text{ km}}$$