This semester: Light (e/m radiation)
Relativity
Quantum physics, including atomic and nuclear physics

Chapter 33 Electromagnetic (e/m) waves —

OH — J C Maxwell

Recall from Phys 4B: $\vec{E}$ = electric field, $\vec{B}$ = magnetic field } in empty space

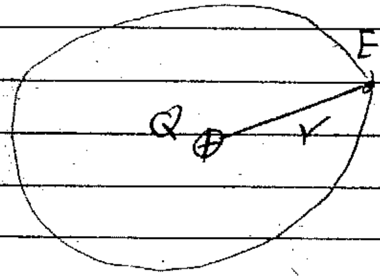
Maxwell's equations — provide a complete description of electricity, magnetism, and light (new this semester)

Gauss's law for $\vec{E}$ fields (Chapter 22-2 of Tipler & Mosca, Ch. 21 of Wolfson)

(1) $\oint \underbrace{\vec{E}\cdot d\vec{A}}_{}$ (surface integral over area A) $= \dfrac{Q}{\varepsilon_0}$ (van de Graaf demo)

$\Rightarrow$ properties of the E field

Example — Find E at distance r $[E = E(r)]$ from a point charge Q:



$E \cdot 4\pi r^2 = \dfrac{Q}{\varepsilon_0}$ since $A_{sphere} = 4\pi r^2$

$\Rightarrow E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2} = k_e\dfrac{Q}{r^2}$, $k_e = \dfrac{1}{4\pi\varepsilon_0}$

with $\vec{F}_{coulomb} = q\vec{E}$ $\boxed{\varepsilon_0 = 8.854\times10^{-12}\ \dfrac{C^2}{Nm^2}}$

$\Rightarrow \vec{F}_{coulomb} = k_e\dfrac{qQ}{r^2}$, Coulomb's law.

Most of Phys 4B can be similarly derived from Maxwell's eq.s

Gauss's law for $\vec{B}$ fields (Chapter 27-3 of Tipler & Mosca, Ch. 26 of Wolfson)

(2) $\oint \vec{B}\cdot d\vec{A} = 0$ (Magnets)

$\Rightarrow$ There are no isolated magnetic charges (monopoles): all magnets have both north and south poles.

Faraday's law (Chapter 28-2 of T&M, Ch. 26 of Wolfson)

(3) $\oint \vec{E}\cdot d\vec{s}$ (line integral around perimeter of length s) $= -\dfrac{d\Phi_B}{dt}$, where $\underbrace{\Phi_B}_{\text{magnetic flux}} = \int \vec{B}\cdot d\vec{A}$ where A = area of the loop with perimeter S.

$\Rightarrow$ A time-variable magnetic field (from e.g. a rotating magnet) makes (induces) a current in a loop of wire $\Rightarrow$ Electric generators

$\Rightarrow$ A magnetic field is equivalent to a loop of current

André-Marie Ampère's law (Chapter 27-4 of T&M, Ch. 26 of Wolfson)

(4) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \underbrace{\mu_0\varepsilon_0\dfrac{d\Phi_E}{dt}}_{\text{displacement current}}$, where $\underbrace{\Phi_E}_{\text{electric flux}} = \int \vec{E}\cdot d\vec{A}$ and $\mu_0 = 4\pi\times10^{-7}\ Tm/A$

$\Rightarrow$ A time-variable electric field (from a current in a wire, since current $I = \dfrac{dQ}{dt}$) generates a magnetic field $\Rightarrow$ Electric motors (the opposite of electric generators)

How Maxwell showed light is electromagnetic (e/m) waves — (2)

Observed:

$\vec{E}$ and $\vec{B}$ fields can travel (or propagate) through empty space.

In empty space, containing no sources, $\underset{\text{charge}}{Q} = \underset{\text{current}}{I} = 0$ $\left(\text{since } I = \frac{dQ}{dt}\right)$

In empty space,

Faraday's law simplified, $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)$

this becomes: $E \cdot dx = -\frac{d}{dt}(B \cdot \underbrace{dx \cdot dx}_{= dA,\ \text{since over an area.}})$

$$\Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \quad (33.15)$$

For Ampère's law in emty space, displacement current $\mu_0 I = 0$,

so: $\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$

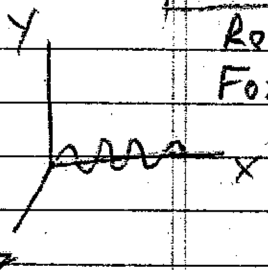
$$\Rightarrow \frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \quad (33.18)$$

This minus sign should be here: see the rigorous derivation on pp. 879-880 of Serway

Combine eq.s. 33.15 and 33.18 (try it!) and one can get:

$$\boxed{\frac{\partial^2 E}{\partial t^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad (33.19)$$

$$\boxed{\frac{\partial^2 B}{\partial t^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}} \quad (33.20)$$



Recall from Chapter 14: These are wave equations.

For any wave traveling along the x-axis:

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{1}{v^2}\right) \frac{\partial^2 y}{\partial t^2}$$ where y = amplitude of the wave and v = speed of the wave, in the $+x$ direction.

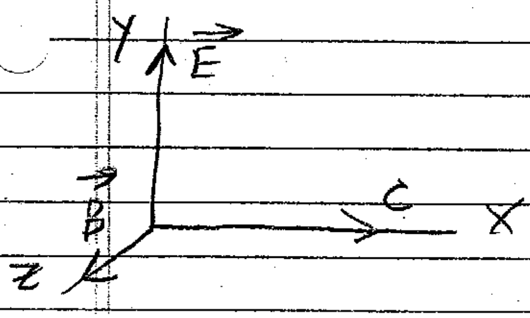
For e/m waves, $\frac{1}{v^2} = \mu_0 \varepsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\boxed{= 3.00 \times 10^8 \frac{m}{s} = c,\ \text{the speed of light in empty space}}$

(since $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$, $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/N \cdot m^2$)

wire demo

$\Rightarrow$ Light is an electromagnetic wave with speed c in empty space (in a vacuum).

[$c = 2.99792 \times 10^8\ m/s$]



Show off — e/m wave /

do silly dance — demo with XYZ wire demo

$$\boxed{\begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad (33\text{-}19) \\ \frac{\partial^2 B}{\partial x^2} &= \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \quad (33.20) \end{aligned}}$$

Review:
Chapter 14 (waves)

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These are wave equations. The simplest solutions to them are:

$E = E_{max} \cos(kx - \omega t)$

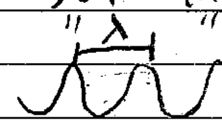
and $B = B_{max} \cos(kx - \omega t)$ (Try it!)

demo – motor-sticks

These are plane waves,

where E_{max} = maximum value for the E field

B_{max} = " " " " B "



λ = wavelength (m)

k = wave number $= \frac{2\pi}{\lambda}$ (m^{-1} or $\frac{\text{waves}}{m}$)

f = frequency (Hertz or Hz, where $1\ Hz = 1\ s^{-1} = 1\ \frac{\text{cycle}}{s}$)

For visible light, $\lambda \sim 4\text{-}7.5 \times 10^{-7}$ m and $f \sim 6 \times 10^{14}$ Hz.

(Every second, $\sim 6 \times 10^{14}$ light waves go into your eyes.)

From Chapter 14, recall that: $\lambda f = v$ for any wave.

For e/m radiation (light) in empty space, $v = c$,

so: $\lambda f = c$ in empty space.

Angular frequency $\omega \equiv 2\pi f$.

Notice that: $\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$. ✓

Also, since $E = E_{max} \cos(kx - \omega t)$

and $B = B_{max} \cos(kx - \omega t)$,

and $\partial E/\partial x = -\partial B/\partial t$,

$$\frac{\partial}{\partial x}\left(E_{max} \cos[kx - \omega t]\right) = -\frac{\partial}{\partial t}\left(B_{max} \cos[kx - \omega t]\right)$$

$$-k E_{max} \cancel{\sin(kx - \omega t)} = -\omega B_{max} \cancel{\sin(kx - \omega t)}$$

$$\Rightarrow \frac{E_{max}}{B_{max}} = \frac{\omega}{k} = c$$

$\therefore$ $\boxed{\frac{E}{B} = \frac{E_{max}}{B_{max}} = c}$ for e/m waves in empty space.

Notice: $\vec{E}$ and $\vec{B}$ will be in phase.

OH & handout – Heinrich Hertz & Wilhelm Röntgen

The electromagnetic spectrum

Visible and invisible light are electromagnetic radiation, or the e/m spectrum.

Visible light, which human eyes can see, makes up only a tiny portion of the possible kinds of e/m radiation, the difference between which is their wavelength:

Region of the spectrum	Wavelength (λ)	Typical Sources
Gamma rays	Short!	Nuclear reactions
X-rays	10^{-10} m	Inner-electron transitions
Ultraviolet radiation	0.01 – 0.4 microns (1 micron = 10^{-6} m)	The hottest stars
Visible light	0.4 – 0.78 microns	Outer-electron transitions, and the Sun
Infrared Radiation	0.78-100 microns	Room-temperature objects, including human bodies and planets
Microwaves	millimeters	
Radio waves	centimeters & longer	The very cold gas between the stars

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The properties of electromagnetic (e/m) radiation

In 1887, Heinrich Hertz experimentally confirmed Maxwell's electromagnetic theory, by being first to transmit and receive radio waves.

In 1895, Wilhelm Röntgen serendipitously (accidentally) discovered X-rays.

They were named X-rays because their nature was unknown, at first. (The symbol X denotes an unknown.)

In a series of experiments, Hertz showed that radio waves are e/m radiation, with wavelengths over 1000 times *longer* than visible light.

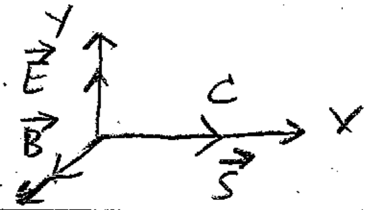
In a similar series of experiments, Röntgen showed that X-rays are also e/m radiation, with wavelengths 1000 times *shorter* than visible light.

Both series of experiments did this by showing that radio waves and X-rays have these properties:

- They have speed $v = c$, the speed of light, in empty space.
- They are *transverse* waves, with $\vec{E} \perp \vec{B} \perp$ the direction of travel.
- They have $E/B = E_{max}/B_{max} = c$ in empty space.
- They *reflect* (Chapters 34 and 35).
- They *refract* (Chapters 34 and 35).
- They *interfere* with other waves (Chapter 36).
- They *diffract* (which means bend around corners) (Chapter 37).
- They can be *polarized* (which means they can have E fields only in a specific direction) (Chapter 37).

Visible light does all this too!

So do all other forms of e/m radiation, including infrared and ultraviolet radiation, microwaves, etc.



(6)

John Henry Poynting
1852–1914
Student of Maxwell's

E/M waves carry energy —

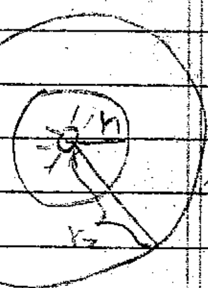
The energy flux (or flow) is given by the Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \text{Units: } W/m^2 \quad (\text{Power}/\text{Area})$$

Example — At Earth, we get an average $|\vec{S}|_{avg} = S_{avg} = I$ (intensity)
$= 1340\ W/m^2$ of sunlight, from the Sun. (The text says "$\approx 1{,}000\ \frac{W}{m^2}$")

For plane waves, $|\vec{E}\times\vec{B}| = EB$,

so $|\vec{S}| = \frac{EB}{\mu_0} = \frac{E_{max}^2}{\mu_0 c} = \frac{B_{max}^2 c}{\mu_0}$ since $\frac{E}{B} = \frac{E_{max}}{B_{max}} = c$



Also, Intensity (W/m^2) $= I = \frac{E_{avg}^2}{\mu_0 c} = \frac{E_{max}^2}{2\mu_0 c}$ since $\langle\cos^2(kx-\omega t)\rangle = \frac{1}{2}$

$P_{av} = \frac{1}{T}\int_0^T P\,dt$

so $\frac{1}{2\pi}\int_0^{2\pi}\cos^2(kx-\omega t)\,dt = \frac{1}{2}$ ✓

I falls off with r^2,

since Power $= I_1 A_1 = I_2 A_2$
$= I_1 4\pi r_1^2 = I_2 4\pi r_2^2$

so $\frac{I_1}{I_2} = \frac{4\pi r_2^2}{4\pi r_1^2}$ and $\boxed{\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2}$

Example — Cassini spacecraft at Saturn
$r = 10\ AU$
$r_{Earth} = 1\ AU$
$\frac{I_{Saturn}}{I_{Earth}} = \left(\frac{r_{Earth}}{r_{Saturn}}\right)^2 = \left(\frac{1}{10}\right)^2$
$I_{Saturn} = \frac{1}{100} I_{Earth} = \frac{1340\ W/m^2}{100}$
$= 13.4\ W/m^2$ ✓

$\rho(\text{air}) = 1.225\times10^{-3}\ \frac{g}{cm^3}$

$n(\text{air}) = 2.5\times10^{19}\ \frac{\text{molecules}}{cm^3}$

$\rho(H_2O) = 1\ g/cm^3$

$\rho(Fe) = 7\ \frac{g}{cm^3}$

$\rho(Pb) = 11\ \frac{g}{cm^3}$

Energy density — Joules/m^3

Recall from Phys 4B:

Electric energy density $u_E = \frac{1}{2}\varepsilon_0 E^2$ (33-28) (25.15)

Magnetic energy density $u_B = \frac{1}{2\mu_0}B^2$ (33-29) (31-14)

For an e/m wave,

$$u_E = u_B = \frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0} \quad \text{since } \frac{E}{B} = c \text{ and } \frac{1}{c^2} = \mu_0\varepsilon_0$$

Total instantaneous energy density

$$u = u_E + u_B = \varepsilon_0 E^2 = B^2/\mu_0$$

Total average energy density

$$(u_{total})_{avg} = \varepsilon_0 (E^2)_{avg} = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0}, \text{ since } \langle\cos^2(kx-\omega t)\rangle = \tfrac{1}{2}.$$

$I = |\vec{S}|_{avg} = (u_{total})\,c \;\Rightarrow$ Energy density travels with speed c, in empty space.

Units for energy density:

$$u_{Avg} = S_{avg}/c = \frac{(W/m^2)}{m/s} = \frac{W\cdot s}{m^3} = \frac{J}{m^3} \quad \left(\text{since } 1\text{ watt} = 1\ \frac{\text{Joule}}{s}\right)$$

E/M waves carry momentum —

Radiation pressure $P = |\vec{S}|/c$ for 100% absorption (black surface)

$P = 2|\vec{S}|/c$ for 100% reflection (white surface)

Since $|\vec{S}| = \frac{1}{\mu_0}|\vec{E} \times \vec{B}| = \frac{W}{m^2}$,

$P = \left(\frac{W}{m^2}\right) / \left(\frac{m}{s}\right) = \frac{\text{Newtons}}{\text{Area}} = \text{Pressure}$ ✓

OH — Comet tails

OH — solar sail