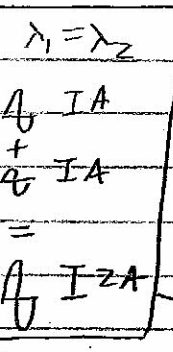
Demo – laser two-slit interference

Demo – two wavy 2-m sticks

Demo – superposition

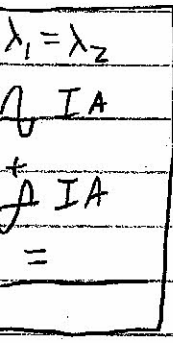
Chapter 36 – Interference of waves (wave optics) –



From Phys 4A (Chapters 13-14), recall the superposition principle: waves add (or cancel) linearly, in amplitude and in phase.

Light waves, or any waves, combine in two ways:

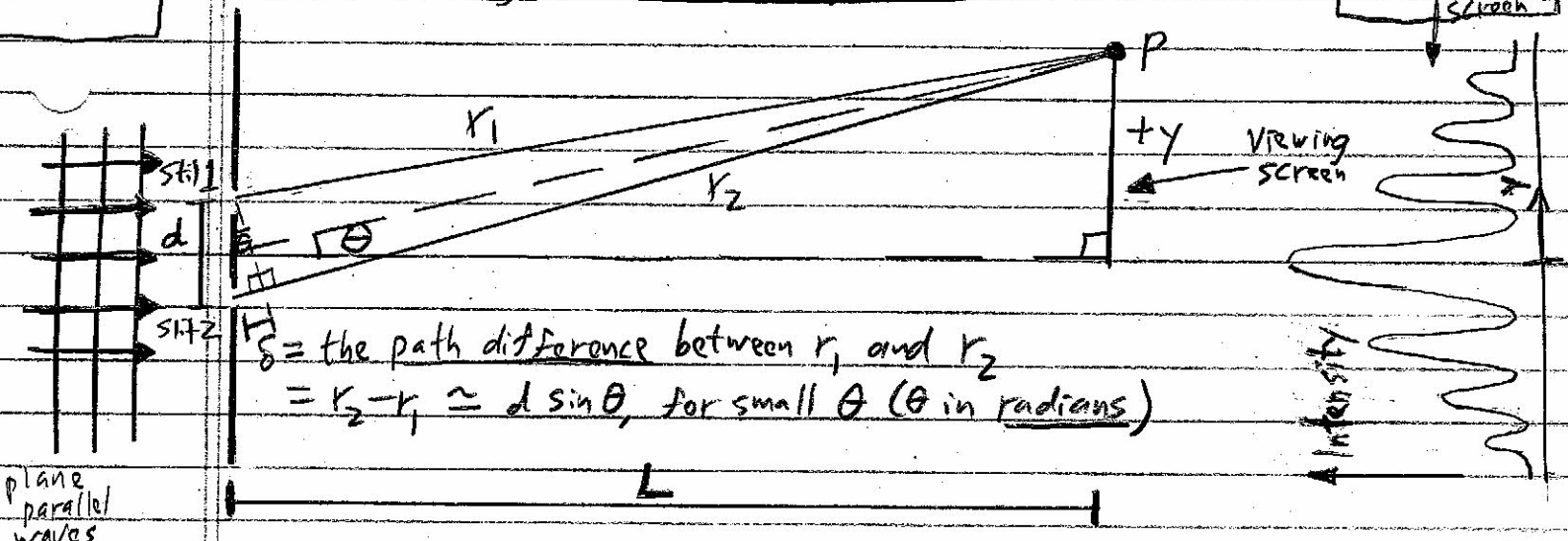
(1) Constructive interference, when waves add in phase, to make bright fringes (for light)



(2) Destructive interference, when waves cancel when out of phase, making dark fringes.

Fringes are patterns of light and dark bands.

Thomas Young's Two-Slit Experiment (1801) –



δ = the path difference between r_1 and r_2
$= r_2 - r_1 \simeq d \sin\theta$, for small θ (θ in radians)

Constructive interference (making a bright spot on the viewing screen) occurs when:

$$d \sin\theta \simeq \delta = m\lambda, \quad m = 0, \pm 1, \pm 2, \ldots \qquad (1)$$

Destructive interference (making a dark spot on the viewing screen) occurs when:

$$d \sin\theta \simeq \delta = \left(m + \tfrac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \ldots \qquad (2)$$

Show figure – overlapping waves (Moiré pattern)

m = the order of the fringe.
m = ±1 is first order,
m = ±2 is second order, etc.

show table — the small angle approximation

Notice that, for small θ (where θ is in radians),
$\sin\theta \simeq \theta \simeq \tan\theta$.
This is called "the small angle approximation."
It's OK for $\theta \lesssim 45°$: recall the simple pendulum from Phys 4A.

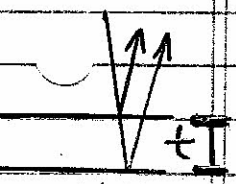
Since $\sin\theta \simeq \tan\theta = y/L$ (for small θ, θ in radians),
equations (1) and (2) above become:

$y_{bright} \simeq \frac{\lambda L}{d} m$ for constructive interference
(where the bright fringes are, on the viewing screen)

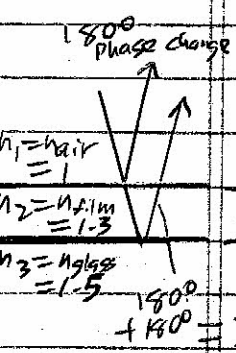
$y_{dark} \simeq \frac{\lambda L}{d}(m+\frac{1}{2})$ for destructive interference
(where the dark " " , " " " " ")

Interference by thin films –

Applications: optical filters, anti-reflection coatings for optics,
"stealth" anti-radar technology.
Phenomena: colors in soap bubbles and oil slicks, iridescence.

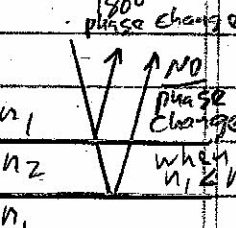


For a nearly transparent, thin coating on a surface,
viewed at nearly normal incidence (almost straight down),
the path difference $\delta \simeq 2t$, where t = the thickness of the film.
For constructive interference, $\delta = m\lambda$, so $2t = m\lambda_{film} = \frac{m\lambda}{n_{film}}$ $\left(\text{since } n_{film} = \frac{\lambda}{\lambda_{film}}\right)$

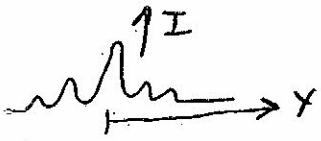


For an anti-reflection coating on a lens (air–film–glass),
with $n_1 < n_2 < n_3$,
$2n_{film}t = m\lambda$ constructive interference,
$2n_{film}t = (m+\frac{1}{2})\lambda$ destructive interference
→ There is a 180° phase change at the top surface,
and another (therefore none) at the bottom surface.

$n_1 = n_{air} = 1$
$n_2 = n_{film} = 1\text{-}3$
$n_3 = n_{glass} = 1\text{-}5$



For a soap bubble (air–film–air), it's reversed –
$2n_{film}t = (m+\frac{1}{2})\lambda$ constructive interference,
$2n_{film}t = m\lambda$ destructive interference.
→ This is because there is a phase change
at the top surface, but not at the bottom surface.



bright
dark
bright
dark
bright

The Intensity Pattern on the Viewing Screen –

For the light wave through slit 1, $E_1 = E_{max} \sin(\omega t)$

" " " " " slit 2, $E_2 = E_{max} \sin(\omega t + \phi)$

Constructive interference occurs when $\phi = 2\pi = 360°$.

Destructive " " " $\phi = \pi = 180°$.

From superposition, $E_{total} = E_1 + E_2$

$= E_{max}[\sin(\omega t) + \sin(\omega t + \phi)]$

Recall the trig identity:

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right).$$

Let $A = \omega t + \phi$ and $B = \omega t$.

Then:

$$E_{total} = 2E_{max}\left[\cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)\right].$$

Intensity $I \propto E_{total}^2 = 4E_{max}^2\left[\cos^2(\phi/2) \sin^2\left(\omega t + \frac{\phi}{2}\right)\right]$.

Take the time average, with $\langle \sin^2\theta \rangle = 1/2$

↑ Average, between 0 and 2π.

$$\Rightarrow I_{avg} = \frac{1}{2} I_{max} \cos^2(\phi/2).$$

Recall the path difference $\delta = r_2 - r_1 \simeq d \sin\theta$.

When $\delta = \lambda$, constructive interference occurs,

so $\frac{\delta}{\phi} = \frac{\lambda}{2\pi}$

$$\Rightarrow \boxed{I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

Also, since $\sin\theta \simeq \tan\theta = y/L$ for small θ,

$$\boxed{I = I_{max} \cos^2\left(\frac{\pi d y}{\lambda L}\right)},$$

which is constructive interference at:

$y_{bright} = (\lambda L/d)\, m$. ✓