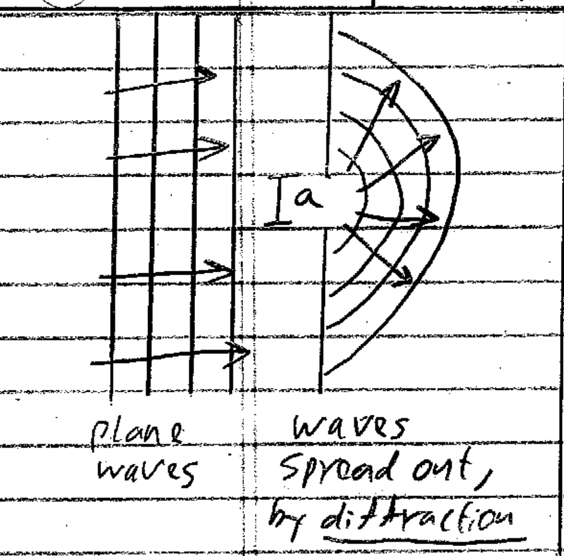
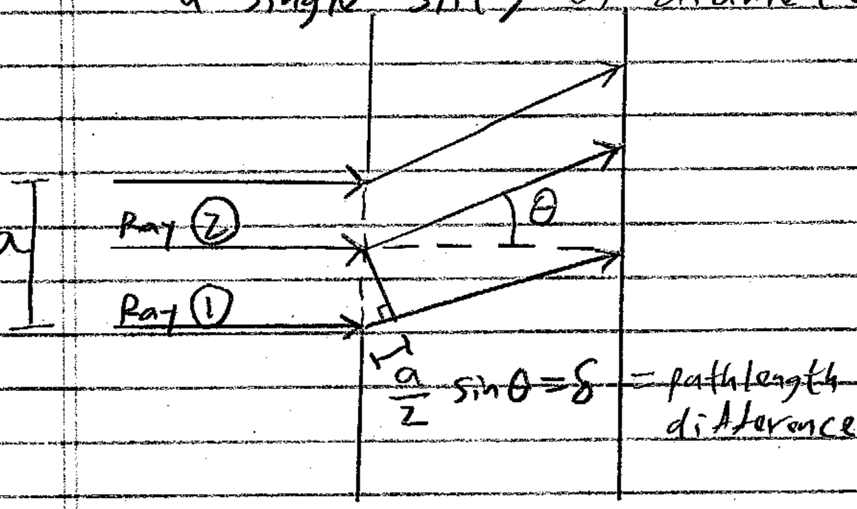
Chapter 37 — Diffraction = bending of a wave around an obstacle, or through a gap, or aperture, of diameter a.

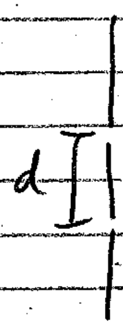


→ Related to interference

(You can hear around corners, but not see around corners, not just because of echoes [reflection], but mainly because of diffraction, and also $\lambda(\text{sound}) > \lambda(\text{light})$.)

Fraunhofer diffraction = pattern caused by a single slit, of diameter a.





Recall that destructive interference, for two slits (separated by d) occurs when:

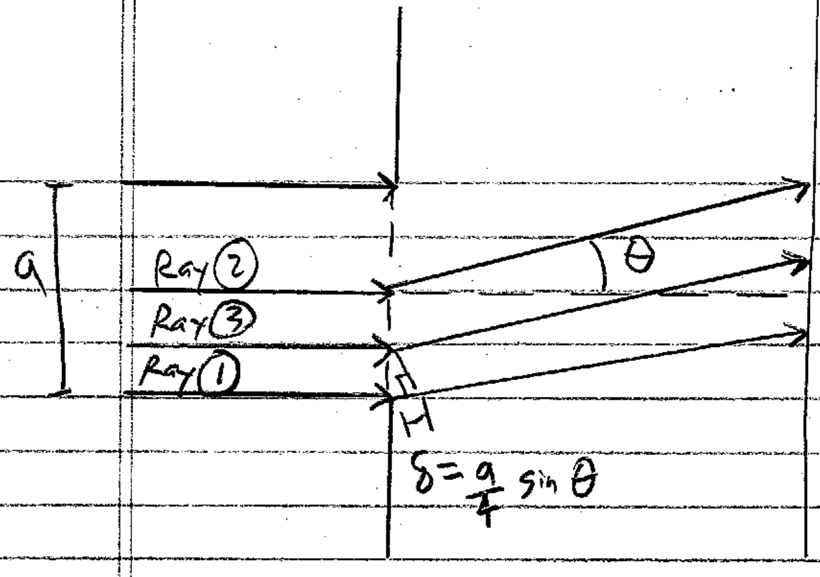
$$\delta = d \sin\theta = (m + \tfrac{1}{2})\lambda, \quad m = 0, \pm 1, \pm 2, \ldots$$

so: $d \sin\theta = \frac{\lambda}{2}$, when $m = 0$ in the center of the pattern.

For Fraunhofer diffraction, destructive interference between Ray (1) and Ray (2) occurs when:

$$\delta = \frac{a}{2}\sin\theta = (m + \tfrac{\lambda}{2}) = \frac{\lambda}{2} \text{ for } m = 0,$$

so: $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ and $\boxed{\sin\theta = \lambda/a}$



Now consider Ray (1) and Ray (3), where Ray (3) is halfway between Ray (1) and Ray (2).

Destructive interference between Ray (1) and Ray (3) occurs when:

$$\delta = \frac{a}{4}\sin\theta = (m+\tfrac{1}{2})\lambda = \frac{\lambda}{2},$$

for $m=0$,

so: $$\boxed{\sin\theta = \frac{2\lambda}{a}}.$$

Likewise for $\sin\theta = \frac{3\lambda}{a}$, etc. (Proof by induction)

So in general for Fraunhofer diffraction, the zero intensity (dark fringes) occurs when:

$$\boxed{a\sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots}$$

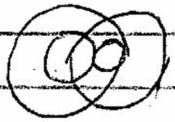
The maxima are roughly halfway in between.

Resolving power —

Diffraction smears images, making it difficult to resolve two light sources as separate.

Examples —

- 2 car taillights appear to converge in the distance.
- Even Hubble Space Telescope can't get unlimited resolution.
- Why ordinary microscopes (that use visible light) can't see atoms



The Rayleigh criterion — "The diffraction limit" ("Diffraction-limited optics")

To resolve two sources, the first diffraction minimum lies on the next maximum.

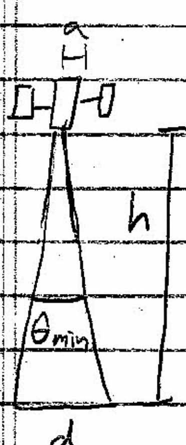
$\Rightarrow \sin\theta_{min} = \dfrac{\lambda}{a}$ for a slit (rectangular).

For a circular aperture, as most telescopes, microscopes, cameras, and lenses have:

$$\boxed{\sin\theta_{min} \simeq \frac{1.22\lambda}{a}}$$

(The 1.22 comes from a nasty derivation, which I will (mercifully) skip.)

Example — Why a satellite looking at Earth can't read a wristwatch —



Suppose $a = 2.4$ m (a KH-11 spy satellite).

$\lambda = 550$ nm (visible light)

h = altitude of satellite above Earth $\simeq 200$ km

d = size of smallest discernible object

so: $\sin\theta_{min} \simeq \theta_{min}$ for small θ_{min}, θ_{min} in radians,

and: $\dfrac{d}{h} \sim \theta_{min} \simeq \dfrac{1.22\lambda}{a}$,

so: $d = 1.22\lambda h/a = (1.22)(5.5\times10^{-7}\text{m})(2\times10^{5}\text{m})/2.4\text{m}$

$\boxed{d = 6 \text{ cm}}$ so a satellite can't read a watch.

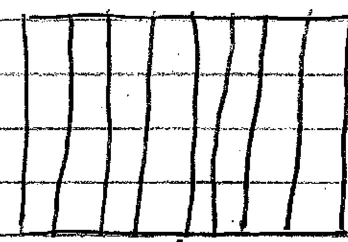
Notice how diffraction depends on wavelength (λ).

⇒ One can use diffraction (and interference) to break light into colors, as with refraction (although the colors are in reversed order).

Diffraction gratings are a "super" Young's experiment, with thousands of slits.

Example – A CD or DVD, which breaks light into colors, due to its slits.

As before, $d \sin\theta = m\lambda$, but here, d = distance between the slits (assuming all are the same).



If $N = \frac{\text{total number of slits}}{\text{length}}$, e.g. $300 \frac{\text{lines}}{\text{mm}}$,

then $d = 1/N$.

Can use as a prism:

$\sin\theta_{red} = \lambda_{red}/d$
$\sin\theta_{violet} = \lambda_{violet}/d$ } for $m=1$.

Notice, though: the colors are reversed. Red light is diffracted (bent) more, but violet light is refracted (bent) more.
A grism is a grating and a prism, to disperse colors more.

Spectral resolution $R = \frac{\lambda_{average}}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda} = Nm$

↑ You get more than one spectrum with a grating.

= how far the wavelengths spread out, by a grating or a prism (or both).