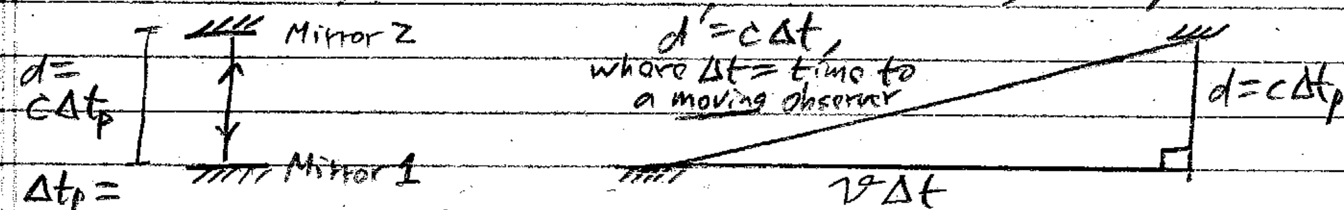
Chapter 38 (continued) — Einstein's Special Theory of Relativity

In 1905, Albert Einstein wrote 4 papers: (a) Special relativity
(b) $E=mc^2$ (next class)
(c) The photoelectric effect (Chapter 39) – Nobel prize, 1921.
(d) Brownian motion (proof that molecules exist)

The Special Theory of Relativity (SR) —
Assumes only:
(1) c is constant in all inertial reference frames
(2) The Principle of Relativity — the laws of physics are the same in all inertial reference frames.

The light clock — is a clock made by bouncing a pulse of light between two mirrors.

Stationary observer (rest frame) | Moving observer (moving frame)



Δt_p = proper time = time to a stationary observer

Notice this is a right triangle.
Since $c^2 = a^2 + b^2$ for a right triangle,

$$(c\Delta t)^2 = (c\Delta t_p)^2 + (v\Delta t)^2$$
$$c^2\Delta t^2 = c^2\Delta t_p^2 + v^2\Delta t^2$$
$$(c^2 - v^2)\Delta t^2 = c^2\Delta t_p^2$$

$$\Rightarrow \frac{\Delta t_{moving}}{\Delta t_{rest}} = \frac{\Delta t}{\Delta t_p} = \frac{1}{\sqrt{1-(v/c)^2}} \equiv \gamma, \text{ the time dilation factor}$$

→ Time goes slower as you travel faster! (well documented now)

Time dilation —

v/c	γ
0.1	1.005
0.5	1.155
0.8	1.67
0.9	2.29
0.99	7.09
0.999	22.4

$\gamma = 1$ for $v = 0$ only
$\gamma > 1$ for $v > 0$.
$\gamma \to \infty$ as $v \to c$, so c is the speed limit of the Universe. Energy, matter, and information can't have $v > c$.

Everything is a function of time!

Consequences of time dilation —

Length contraction — $L_{moving} = L_{rest}\sqrt{1-(v/c)^2}$

or: $L = L_p / \gamma$,

where L_p = proper length (the length at rest)

This is because:

$\Delta t = \gamma(\Delta t) = \frac{L_p}{c}\frac{1}{\sqrt{1-(v/c)^2}}$ since time = $\frac{\text{distance}}{\text{speed}}$.

Galilean transformation (from one inertial frame to another) (assumed by Newton)

$x' = x - vt$
$y' = y$
$z' = z$
$t' = t$ ← only an assumption!

Lorentz transformation (correct form, for SR)

$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

⇒ Relativistic velocity transformations — addition of relativistic velocities.

In Newtonian physics, velocities add simply.

For example: 90 mph = 60 mph + 30 mph

or: $u_x = u_x' + v$

For relativistic motion: $u_x = \frac{u_x' + v}{\left(1 + \frac{u_x' v}{c^2}\right)}$

Example — $u_x = \frac{\frac{1}{2}c + \frac{3}{4}c}{1 + \left(\frac{3}{4}c\right)\left(\frac{1}{2}c\right)/c^2}$

$= \frac{10}{11}c < c$

Also: the solution for Michaelson-Morley —

$u_x = \frac{c + v}{\left(1 + \frac{cv}{c^2}\right)} = \frac{c+v}{(c+v)/c} = c$!

So add c to any speed v, and you get c.

Relativistic dynamics – Einstein's 2^{nd} 1905 paper (May)

Newtonian momentum (at $v \ll c$) becomes relativistic momentum, at $v \to c$.

$$\vec{p} \equiv m\vec{v} \longrightarrow \vec{p} \equiv \gamma m \vec{v} \rightarrow$$

$\boxed{\text{Einstein had to redefine momentum, so it's still conserved. Notice that } \gamma \to 1 \text{ at } v \ll c.}$

Force $\vec{F} = \dfrac{d\vec{p}}{dt} \longrightarrow \vec{F} = \dfrac{d\vec{p}}{dt}$ unchanged, provided $\vec{p} = \gamma m \vec{v}$.

Kinetic energy $KE = \frac{1}{2}mv^2 = \dfrac{p^2}{2m} \longrightarrow KE = E - mc^2 = (\gamma - 1)mc^2$ (derived below)

From Phys 4A, recall that Work $W = \displaystyle\int_{x_1}^{x_2} \vec{F}\cdot d\vec{x} = \int_{x_1}^{x_2} \left(\frac{d\vec{p}}{dt}\right)\cdot d\vec{x}$

$$\left(\frac{dp}{dt}\right)dx = \underbrace{\frac{d}{dt}(\gamma m v)\,dx}_{\text{since } p = \gamma m v} = \underbrace{\frac{d}{dt}(\gamma m v)\,v\,dt}_{\text{since } v = dx/dt} = \frac{d}{dt}\left[\frac{mv}{\sqrt{1-(v/c)^2}}\right]v\,dt$$

$$W = \int_{x_1}^{x_2} v\,dt\left[\frac{m\,(dv/dt)}{(1-v^2/c^2)^{3/2}}\right] = \int_0^{v_f} \frac{mv\,dv}{(1-v^2/c^2)^{3/2}}$$

start from rest ($v_i = 0$) and change variables.

From Phys 4A, recall that $W = KE_f - KE_i = KE_f$, if $v_i = 0$.

solve integral $\rightarrow W = KE_f = \dfrac{mc^2}{\sqrt{1-(v_f/c)^2}} - mc^2 = (\gamma - 1)mc^2$, as stated above.

$$E_{total} = E = KE + mc^2 = \frac{mc^2}{\sqrt{1-(v/c)^2}} = \gamma mc^2 \Rightarrow \boxed{E_{total} = \gamma mc^2}$$

Notice that $E_{total} \to \infty$ as $v \to c$, so $v \neq c$.

For an object at rest, $v = 0$ and $\gamma = 1$,

So that: $\boxed{E = mc^2} \rightarrow \boxed{\text{Mass and energy are equivalent!}}$

Also: since $p = \gamma m v \Rightarrow E^2 = p^2c^2 + (mc^2)^2$ (lots of algebra)

For photons (light particles, with $m = 0$ and $v = c$), $E = pc$ (Useful!)

New energy units: 1 electron-volt $= 1\,eV = 1.602 \times 10^{-19}\,J$

Recall $U = qV$, here, $q = e$ and $V = 1$ volt.

proton mass $m_p = 1.67 \times 10^{-27}$ kg $= 938.3\ MeV/c^2$.

$1\,keV = 10^3\,eV$

$1\,MeV = 10^6\,eV$.

Electron mass $m_e = (1/1836)m_p = 0.511\ MeV/c^2 = 511\ keV/c^2$.

Example 1 – Matter/Antimatter annihilation –

proton $\oplus$, Anti-proton $\ominus$ $\rightarrow$ 938.3 MeV γ-ray photon + 938.3 MeV γ-ray photon.

Example 2 – Pair production ($E = mc^2$ in reverse)

0.511 MeV γ-ray + 0.511 MeV γ-ray $\rightarrow$ $\ominus$ electron, with $m_e = 0.511\ MeV/c^2$. + $\oplus$ positron (anti-electron), with $m = m_e$.