Chapter 39 – Quantum physics of particles and waves –

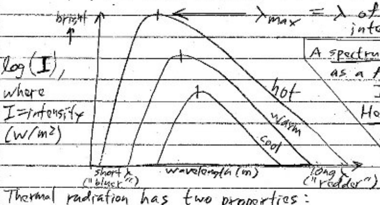
"Quantum" means "particle" in Latin.

Special relativity replaces Newtonian physics at $v \to c$.

Quantum physics replaces Newtonian physics at $r \lesssim r(\text{molecule})$ (anything smaller than a molecule).

Thermal radiation – is the e/m radiation given off by hot objects. This is also called blackbody radiation, since an object at $T = 0K$ would be perfectly black.

Anything opaque (not transparent to light) with $T > 0K$ (namely, everything) gives off e/m radiation with a distinctive spectrum, which depends on its temperature.



A spectrum is intensity I as a function of wavelength λ, $I = I(\lambda)$.
Here, though, it's also a function of temperature: $I = I(\lambda, T)$.

Thermal radiation has two properties:

(1) Wien's law – $\lambda_{max}(m) = \dfrac{\text{constant}}{T(K)} = \dfrac{2.898 \times 10^{-3}\ m\cdot K}{T(K)}$

In other words, hotter objects look bluer, and cooler objects look redder (e.g. flames, stars, hot pieces of metal).

Examples –

(a) The Sun has $T = 5800\ K \simeq 11{,}000°F$.
$\Rightarrow \lambda_{max} = 0.5$ microns $= 5 \times 10^{-7}\ m = 500\ nm$, visible light.
The human eye is most sensitive at $\lambda \simeq \lambda_{max}(\text{Sun})$.

(b) A human body has $T = 98.6°F = 37°C = 310K$.
$\Rightarrow \lambda_{max} = 10$ microns, infrared radiation.
A live human body radiates about 100 W of power at infrared wavelengths, one's body heat.

The other property of thermal radiation —
(2) Stefan's law — $I = \sigma T^4$,
where I = intensity at all wavelengths,
T = temperature (Kelvins),
and sigma σ = the Stefan-Boltzmann constant
$= 5.670 \times 10^{-8} \frac{W}{m^2 T^4}$

Don't confuse this with Boltzmann's constant $k_B = 1.381 \times 10^{-23}$ J/K.

Thermal radiation is a common phenomenon. It is the most common source of visible light, and the most common source of e/m radiation at all wavelengths. (It is by no means the only source, though.)

Classical physics couldn't explain thermal radiation. One attempt was: the Rayleigh-Jeans law:

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$

(skip the derivation)

This works at long λ (for radio), but it predicts too much short-λ radiation: "the ultraviolet catastrophe."

In 1900, Max Planck derived the correct formula for the spectrum of thermal radiation:

$$\boxed{I(\lambda, T) = \frac{2\pi h c^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right]}}$$

(We'll skip the derivation: take Phys 102 if interested.)

Planck did this by assuming that atoms could radiate only at certain frequencies, such that

$E = nhf$, where $n = 1, 2, 3 \ldots$
E = energy radiated (J)
f = frequency (s^{-1})
and h = Planck's constant
$= 6.626 \times 10^{-34}$ J·s

In 1905, Einstein would show that this works because light is a particle, in his paper on the photoelectric effect.

The Photoelectric Effect – discovered by Heinrich Hertz, 1887.

Shining a light on a metal can cause it to emit electrons.

Classical explanation: light is a wave, of intensity I (W/m^2). Electrons absorb this energy, until the binding energy of the metal is exceeded, and so electrons (e^-) are released.

Minimum energy needed to release electrons $\equiv \varphi$, Work Function.

e.g. φ (Na) = 2.46 eV
φ (Cu) = 4.70 eV.
(1 eV = 1.602×10^{-19} Joules.)

Problems with the classical explanation:

(1) KE of e^- should be proportional to I, but isn't.
(In fact, KE of e^- is independent of I.)

(2) The effect should be independent of λ of light, but isn't: in fact, no electrons are emitted if $\lambda > \lambda_C$, the cutoff wavelength (or, equivalently, $f < f_c$, the cutoff frequency: recall that $\lambda f = c$).

Example: the detectors in most digital cameras are made of silicon, They can't detect light with $\lambda > 1$ micron, no matter how intense.

(3) Classical physics predicts that the effect should take several seconds to occur, since it will take this much time for the metal to absorb energy from the waves. It does not: the effect is observed to be nearly instantaneous (within 10^{-9} s).

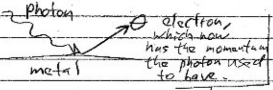
Einstein's explanation for the photoelectric effect (1905, Nobel prize 1921) — Suppose light is a particle, even though Thomas Young's two-slit experiment showed it is a wave.

Photons are particles of light (although Einstein called them "quanta").
Each photon has: $\boxed{E_{photon} = hf = \frac{hc}{\lambda}}$ since $f = \frac{c}{\lambda}$.

Recall from Chapter 38 (on special relativity) that:
$$E^2 = p^2c^2 + (mc^2)^2.$$
photons are light, so $v = c$
and so $m_{photon} = 0$, an odd property.
$$\Rightarrow \boxed{E_{photon} = pc}$$
and so: $\boxed{p_{photon} = \frac{h}{\lambda}}$.

That's right: photons have non-zero energy and momentum, even though they must have zero mass, since $v = c$.

The photoelectric effect therefore becomes easy to explain: it's just a collision between two particles, a photon into the metal knocks an electron out of the metal.



By conservation of energy for this collision:
$$K_{max} = hf - \Phi,$$
where: K_{max} = the maximum kinetic energy of the electron kicked out,
hf = energy of the photon,
Φ = the work function, or binding energy of atoms in the metal.

This solves all 3 problems with the photoelectric effect:
(1) K_{max} is independent of I, as observed.
(2) The cutoff frequency occurs when $K_{max} = 0$, so $hf_c = \varphi$ and so $f_c = \varphi / h$ (observed).
(3) The effect is nearly instantaneous, as observed, since it's a collision between two particles: a photon (coming in), and an electron (kicked out).

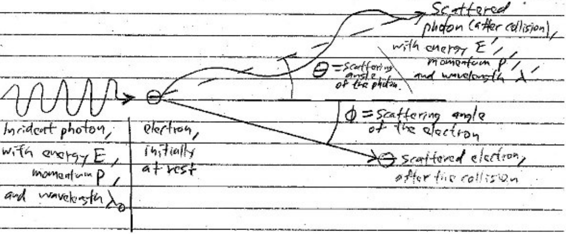
Robert Millikan (1915) experimentally confirmed this (Nobel 1923).

Even more evidence was needed to convince many physicists that photons exist. This was provided by:

The Compton effect (Arthur Holly Compton 1923, Nobel prize 1927) — If light particles (photons) exist, they should scatter, as particles do when colliding with other particles (for example, electrons),

and so: $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1 - \cos\theta)$,

where: λ' = wavelength of the scattered photon (after collision)
λ_0 = wavelength of the incident photon (before the collision)
m_e = electron mass (when an electron is scattered)
θ = scattering angle of the photon, after the collision.



Derivation of the Compton effect -

This is much like an elastic collision problem from Phys 4A, although it does need to be relativistic.

Conservation of energy $\Rightarrow$ $E = E' + K_e$ -
(electron kinetic energy)

$$\Rightarrow \frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + (\gamma - 1) m_e c^2$$

Linear momentum conservation in the x direction $\Rightarrow$

$$\underbrace{p = p' \cos\theta}_{\text{photon}} + \underbrace{p_e \cos\phi}_{\text{electron}}$$

$$\Rightarrow \frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos\theta + \gamma m_e v \cos\phi$$
(for the electron)

Linear momentum conservation in the y direction $\Rightarrow$

$$0 = \frac{h}{\lambda'} \sin\theta - \gamma m_e v \sin\phi$$

since the electron is initially at rest, and the photon initially moves along x.

(for photon)

(for the electron)

(algebra) $\Rightarrow$ $$\boxed{\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)}$$

Here, $\lambda_c \equiv \frac{h}{m_e c}$ = the Compton wavelength for an electron.

Chapter 39.5–39.8 The Principle of Complementarity, or the Wave-Particle Duality of Nature

In 1905, Einstein showed that light has both wave and particle properties.

Photons are particles of light. (Einstein called them "quanta.")

Photons have speed $v = c$ in empty space, so $m_{photon} = 0$.

Also, $E_{photon} = \frac{hc}{\lambda} = pc$, and $p_{photon} = \frac{h}{\lambda}$.

All this was subsequently confirmed by experiment, explaining Planck's solution for the spectrum of thermal radiation, the photoelectric effect and its applications (digital cameras, solar cells, LEDs and flat screens), and the Compton effect.

In 1923, Louis de Broglie argued by analogy that molecules, atoms, electrons and any smaller particles should also have wave properties.

Matter waves have $\boxed{\lambda = \frac{h}{p}}$ just like photons do.

Here, p = momentum.
For the Compton effect, high-energy physics, and astrophysics, p is relativistic since $v \to c$, so $p = \gamma m v$.

For ordinary electrons in atoms, as in (most) chemistry, $v \ll c$, so p is Newtonian, so $p = mv$ works fine.

All this was confirmed by experiment:
electron diffraction and interference (in electron microscopes), as well as atomic and ionic diffraction and interference (in Bose-Einstein condensates).

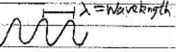
Chapter 39.6–39.8 –

The quantum particle is a particle with wave properties.

We want to find math functions that can describe particles that also have wave properties, for example, anything smaller than a molecule. → These functions are called wave functions, or wavefunctions or eigenfunctions.

Uncertainty relationships for classical waves –

One sine wave has $y = y_1 \sin kx$ where $k = 2\pi/\lambda$.

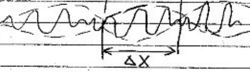


It extends over $-\infty < x < +\infty$, so this wave can't represent a particle, which must be localized.

A better model: add two sine waves together, so:

$y = y_1 \sin k_1 x + y_2 \sin k_2 x$.

If $k_1 \neq k_2$, you get beats.



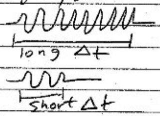
The wave packet is now localized within ΔX, and $\Delta X \Delta k \sim 1$.

⇒ The more precisely we know the particle's position ΔX, the less precisely we know the particle's wavelength Δk, and vice versa.

→ This is an uncertainty relation.

Also: $\Delta \omega \Delta t \sim 1$, where $\omega = 2\pi f$ where f = frequency.

Here, Δt is how long a wave packet lasts:



Heisenberg's Uncertainty Principle –

For matter waves, $\lambda = h/p$.

Also, $k = 2\pi/\lambda$, so: $p = \frac{hk}{2\pi} = \hbar k$ where $\boxed{\hbar = h/2\pi}$

and so: $\Delta p = \hbar \Delta k$.

Since for all waves $\Delta X \Delta k \sim 1$, this implies $\boxed{\Delta X \Delta p_x \geq \hbar/2}$.

As an aside: $\Delta x \Delta p_x \ge \hbar/2$ since $\Delta x \Delta k \sim 1$,
which implies: $(\Delta x)\hbar(\Delta k) \sim \hbar$,
so: $\Delta x \Delta p_x \sim \hbar \ge \hbar/2$,
because it takes at least two points to fit a sinusoid (the Nyquist theorem).

Also, since $E = hf = \hbar\omega$, (Again: $\hbar \equiv \frac{h}{2\pi}$.)
$\Delta E = \hbar \Delta\omega$.
Since for all waves $\Delta\omega\Delta t \sim 1$, this implies $\boxed{\Delta E \Delta t \ge \hbar/2}$.

$$\boxed{\begin{array}{l}\Delta x \Delta p_x \ge \hbar/2 \\ \Delta E \Delta t \ge \hbar/2\end{array}}$$

are Heisenberg's Uncertainty Relationships, referred to together as Heisenberg's Uncertainty Principle (Werner Heisenberg 1927; Nobel Prize 1932).

They imply:
(a) It's not possible to know (or measure) a particle's position (x) and its momentum (p) with unlimited precision at the same time.

(b) The same, for energy (E) and lifetime (t).

→ Major philosophical implications:
departure from Newton's "clockwork" Universe.

(Einstein was wrong when he said "I can't believe God would play dice withe the Universe.")

Example 1 — Absolute zero —
One can't reach $T = 0\,K$, where all molecular motion stops, because the molecules can't stop moving, because of their wave nature.
To reach $T = 0\,K$, $\Delta p = m\Delta v \to 0$
and $\Delta x \to 0$ at the same time.

Example 2 — The transporter unit on Star Trek —
You can't make a whole person disappear and reappear somewhere else, but you can do this with electrons.
If you insist electrons are particles, this is impossible to understand.
It's not so bad if you realize electrons are waves, just sloshing around an obstruction by diffraction.
Electronic devices such as tunnel diodes and Josephson junctions use this!