P34.12 $E = E_{max} \cos(kx - \omega t)$

$$\frac{\partial E}{\partial x} = -E_{max} \sin(kx - \omega t)(k) \quad \to \quad \frac{\partial^2 E}{\partial x^2} = -E_{max} \cos(kx - \omega t)(k^2)$$
$$\frac{\partial E}{\partial t} = -E_{max} \sin(kx - \omega t)(-\omega) \quad \to \quad \frac{\partial^2 E}{\partial t^2} = -E_{max} \cos(kx - \omega t)(-\omega)^2$$

We must show: $\frac{\partial^2 E}{\partial x^2} = \mu_0 \in_0 \frac{\partial^2 E}{\partial t^2}$

That is, $-(k^2) E_{max} \cos(kx - \omega t) = -\mu_0 \in_0 (-\omega)^2 E_{max} \cos(kx - \omega t)$

But this is true, because $\frac{k^2}{\omega^2} = \left(\frac{1}{f\lambda}\right)^2 = \frac{1}{c^2} = \mu_0 \in_0$

The proof for the wave of magnetic field follows precisely the same steps.

P34.13 (a) $f\lambda = c$ or $f(50.0 \text{ m}) = 3.00 \times 10^8 \text{ m/s}$

so $\boxed{f = 6.00 \times 10^6 \text{ Hz} = 6.00 \text{ MHz}}$

(b) $\frac{E_{max}}{B_{max}} = c$ or $\frac{22.0}{B_{max}} = 3.00 \times 10^8 \text{ m/s}$

so $\vec{\mathbf{B}}_{max} = \boxed{-73.3\hat{\mathbf{k}} \text{ nT}}$

(c) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{50.0 \text{ m}} = 0.126 \text{ m}^{-1}$

and $\omega = 2\pi f = 2\pi(6.00 \times 10^6 \text{ s}^{-1}) = 3.77 \times 10^7 \text{ rad/s}$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{max} \cos(kx - \omega t) = \boxed{-73.3\hat{\mathbf{k}} \cos(0.126x - 3.77 \times 10^7 t) \text{ nT}}$$

P34.14 The wave is of the form $E_y = E_{max}\sin(kx - \omega t)$.

(a) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{100 \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 3.33\times10^{-7} \text{ T} = \boxed{0.333\ \mu\text{T}}$

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.00\times10^7 \text{ m}^{-1}} = \boxed{0.628\ \mu\text{m}}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{6.28\times10^{-7} \text{ m}} = \boxed{4.77\times10^{14} \text{ Hz}}$

P34.19 $r = (5.00 \text{ mi})(1\,609 \text{ m/mi}) = 8.04\times10^3 \text{ m}$

$$S = \frac{P}{4\pi r^2} = \frac{250\times10^3 \text{ W}}{4\pi\left(8.04\times10^3 \text{ W}\right)^2} = \boxed{307\ \mu\text{W/m}^2}$$

***P34.20** (a) $\dfrac{P}{\text{area}} = \dfrac{\text{energy}}{\Delta t\cdot\text{area}} = \dfrac{600\times10^3 \text{ Wh}}{(30 \text{ d})(13.0 \text{ m})(9.50 \text{ m})}\left(\dfrac{1 \text{ d}}{24 \text{ h}}\right) = \boxed{6.75 \text{ W/m}^2}$

(b) The car uses gasoline at the rate of $(55 \text{ mi/h})\left(\dfrac{\text{gal}}{25 \text{ mi}}\right)$. Its rate of energy conversion is

$$P = 44.0\times10^6 \text{ J/kg}\left(\frac{2.54 \text{ kg}}{1 \text{ gal}}\right)(55 \text{ mi/h})\left(\frac{\text{gal}}{25 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = 6.83\times10^4 \text{ W}$$

Its power-per-footprint-area is $\dfrac{P}{\text{area}} = \dfrac{6.83\times10^4 \text{ W}}{(2.10 \text{ m})(4.90 \text{ m})} = \boxed{6.64\times10^3 \text{ W/m}^2}$.

(c) A powerful automobile running on sunlight would have to carry on its roof a solar panel huge compared with the size of the car.

(d) Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are current and potential applications of solar energy.

P34.21 Power output = (power input)(efficiency).

Thus, $\text{Power input} = \dfrac{\text{Power out}}{\text{eff}} = \dfrac{1.00\times10^6 \text{ W}}{0.300} = 3.33\times10^6 \text{ W}$

and $A = \dfrac{P}{I} = \dfrac{3.33\times10^6 \text{ W}}{1.00\times10^3 \text{ W/m}^2} = \boxed{3.33\times10^3 \text{ m}^2}$

P34.24 The energy put into the water in each container by electromagnetic radiation can be written as $\Delta E = eP\,\Delta t = eIA\Delta t$ where e is the percentage absorption efficiency. This energy has the same effect as heat in raising the temperature of the water:

$$eIA\Delta t = mc\Delta T = \rho Vc\Delta T$$
$$\Delta T = \frac{eI\ell^2\Delta t}{\rho\ell^3 c} = \frac{eI\Delta t}{\rho\ell c}$$

where ℓ is the edge dimension of the container and c the specific heat of water. For the small container,

$$\Delta T = \frac{0.700\left(25.0\times10^3\ \text{W/m}^2\right)(480\ \text{s})}{\left(10^3\ \text{kg/m}^3\right)(0.0600\ \text{m})(4\,186\ \text{J/kg}\cdot°\,\text{C})} = \boxed{33.4°\ \text{C}}$$

For the larger,

$$\Delta T = \frac{0.910\left(25.0\times10^3\ \text{W/m}^2\right)(480\ \text{s})}{\left(10^3\ \text{kg/m}^3\right)(0.120\ \text{m})(4\,186\ \text{J/}°\,\text{C})} = \boxed{21.7°\ \text{C}}$$

P34.25 (a) $B_{max} = \frac{E_{max}}{c}$: $B_{max} = \frac{7.00\times10^5\ \text{N/C}}{3.00\times10^8\ \text{m/s}} = \boxed{2.33\ \text{mT}}$

(b) $I = \frac{E_{max}^2}{2\mu_0 c}$:

$$I = \frac{\left(7.00\times10^5\ \text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^8\ \text{m/s}\right)} = 6.50\times10^8\ \text{W/m}^2 = \boxed{650\ \text{MW/m}^2}$$

(c) $I = \frac{P}{A}$: $P = IA = \left(6.50\times10^8\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(1.00\times10^{-3}\ \text{m}\right)^2\right] = \boxed{511\ \text{W}}$

P34.27 $S_{av} = \frac{P_{avg}}{4\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$

$$E_{max} = \sqrt{2\mu_0 c S_{av}} = \sqrt{\mu_0 c \frac{P_{avg}}{2\pi r^2}}$$
$$= \sqrt{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^8\ \text{m/s}\right)\frac{4.00\times10^3\ \text{W}}{2\pi\left[4.00(1\,609\ \text{m})\right]^2}} = 0.076\,1\ \text{V/m}$$

The maximum emf (amplitude) induced in a length L of wire is

$$\Delta V_{max} = E_{max}L = (76.1\ \text{mV/m})(0.650\ \text{m}) = \boxed{49.5\ \text{mV}}$$

P34.32 (a) The radiation pressure is

$$P = \frac{2S}{c} = \frac{2I}{c}$$

The force on area A is

$$F = PA = \frac{2\left(1\,370 \text{ W/m}^2\right)}{3.00 \times 10^8 \text{ m/s}}\left(6.00 \times 10^5 \text{ m}^2\right) = \boxed{5.48 \text{ N}}$$

(b) The acceleration is:

$$a = \frac{F}{m} = \frac{5.48 \text{ N}}{6\,000 \text{ kg}} = 9.13 \times 10^{-4} \text{ m/s}^2 = \text{ 913 } \mu\text{m/s}^2 \text{ away from the Sun}$$

(c) It will arrive at time t where $d = \frac{1}{2}at^2$ or

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2\left(3.84 \times 10^8 \text{ m}\right)}{\left(9.13 \times 10^{-4} \text{ m/s}^2\right)}} = 9.17 \times 10^5 \text{ s} = \boxed{10.6 \text{ days}}$$

P34.33 (a) $I = \frac{P}{\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$, and $r = 1.00 \times 10^{-3}$ m:

$$\begin{aligned} E_{max} &= \sqrt{\frac{2\mu_0 cP}{\pi r^2}} \\ &= \sqrt{\frac{2\left[4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right]\left(3.00 \times 10^8 \text{ m/s}\right)\left(15.0 \times 10^{-3} \text{ W}\right)}{\pi\left(1.00 \times 10^{-3} \text{ m}\right)^2}} \\ &= 1.90 \times 10^8 \text{ J} = \boxed{1.90 \text{ kN/C}} \end{aligned}$$

(b) The beam carries power P. The amount of energy ΔE in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c} \quad \rightarrow \quad \Delta E = \frac{P\ell}{c} = \frac{15.0 \times 10^{-3} \text{ W}}{3.00 \times 10^8 \text{ m/s}}(1.00 \text{ m}) = \boxed{50.0 \text{ pJ}}$$

(c) From Equation 34.27 and our result in part (b), the momentum and energy carried a light beam are related by

$$p = \frac{T_{ER}}{c} = \frac{\Delta E}{c} = \frac{50.0 \times 10^{-12} \text{ J}}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.67 \times 10^{-19} \text{ kg}\cdot\text{m/s}}$$

P34.50 $\omega = 2\pi f = 2\pi\left(3.00\times10^{9}\ \text{s}^{-1}\right) = 1.88\times10^{10}\ \text{s}^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 2\pi\left(\frac{3.00\times10^{9}\ \text{s}^{-1}}{3.00\times10^{8}\ \text{m/s}}\right) = 20.0\pi\ \text{m}^{-1} = 62.8\ \text{m}^{-1}$$

$$B_{\text{max}} = \frac{E}{c} = \frac{300\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 1.00\ \mu\text{T}$$

$$\boxed{E = 300\cos\left(62.8x - 1.88\times10^{10}t\right)} \quad \boxed{B = 1.00\cos\left(62.8x - 1.88\times10^{10}t\right)}$$

where *E* is in volts per meter (V/m), *B* is in microtesla (μT), *x* is in meters, and *t* is in seconds.

P34.57 (a) $B_{\text{max}} = \frac{E_{\text{max}}}{c} = \boxed{6.67\times10^{-16}\ \text{T}}$

(b) $S_{\text{av}} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \boxed{5.31\times10^{-17}\ \text{W/m}^2}$

(c) $P = S_{\text{av}}A = \boxed{1.67\times10^{-14}\ \text{W}}$

(d) $F = PA = \left(\frac{S_{\text{av}}}{c}\right)A = \boxed{5.56\times10^{-23}\ \text{N}}$ (approximately the weight of 3 000 hydrogen atoms!)

P34.61 (a) At steady state, $P_{\text{in}} = P_{\text{out}}$ and the power radiated out is $P_{\text{out}} = e\sigma AT^4$.

Thus, $0.900\left(1\,000\ \text{W/m}^2\right)A = 0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)AT^4$

or $T = \left[\frac{900\ \text{W/m}^2}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{388\ \text{K}} = 115°\ \text{C}$

(b) The box of horizontal area *A* presents projected area $A\sin 50.0°$ perpendicular to the sunlight. Then by the same reasoning,

$$0.900\left(1\,000\ \text{W/m}^2\right)A\sin 50.0° = 0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)AT^4$$

or $T = \left[\frac{\left(900\ \text{W/m}^2\right)\sin 50.0°}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{363\ \text{K}} = 90.0°\ \text{C}$

P34.66 We take R to be the planet's distance from its star, and r to be the radius of the planet.

(a) The effective area of the planet over which it absorbs light is its projection onto a plane perpendicular to the light from its sun. The projected area of a planet of radius r is πr^2, so the planet absorbs light over area πr^2.

(b) The planet radiates over its entire surface area, $4\pi r^2$.

(c) At steady-state, $P_{\text{in}} = P_{\text{out}}$: $$eI_{\text{in}}\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$$

$$e\left(\frac{6.00\times10^{23}\ \text{W}}{4\pi R^2}\right)\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$$, so that $6.00\times10^{23}\ \text{W} = 16\pi\sigma R^2 T^4$

$$R = \sqrt{\frac{6.00\times10^{23}\ \text{W}}{16\pi\sigma T^4}}$$

$$= \sqrt{\frac{6.00\times10^{23}\ \text{W}}{16\pi\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(310\ \text{K}\right)^4}} = \boxed{4.77\times10^{9}\ \text{m}}$$