***P35.4** (a) The law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$
$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

This form applies to all kinds of waves that move through space.

In air at 20°C, the speed of sound is 343 m/s. From Table 17.1, the speed of sound in water at 25.0°C is 1493 m/s. The angle is incidence is 13.0°:

$$\frac{\sin 13.0°}{343 \text{ m/s}} = \frac{\sin\theta_2}{1\,493 \text{ m/s}}$$
$$\theta_2 = \boxed{78.3°}$$

(b) The wave keeps constant frequency in all media:

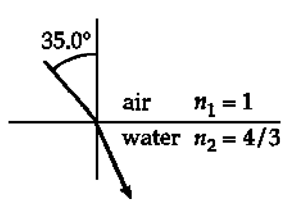
$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$
$$\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{1\,493 \text{ m/s}(0.589 \text{ m})}{343 \text{ m/s}} = \boxed{2.56 \text{ m}}$$

(c) Using Snell's law,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$
$$1.333 \sin\theta_2 = 1.000293 \sin 13.0°$$
$$\theta_2 = \boxed{9.72°}$$

(d) $\lambda_2 = \dfrac{v_2\lambda_1}{v_1} = \dfrac{n_1\lambda_1}{n_2} = \dfrac{1.00293(589 \text{ nm})}{1.333} = \boxed{442 \text{ nm}}$

(e) The light wave slows down as it moves from air to water, but the sound wave speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wave-length increases.



ANS FIG. P35.4

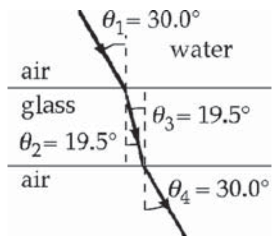
P35.12 $n_1 \sin\theta_1 = n_2 \sin\theta_2$: $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$

$$\theta_2 = \sin^{-1}\left(\frac{1.00 \sin 30°}{1.50}\right) = \boxed{19.5°}$$

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals.

So, $\theta_3 = \theta_2 = \boxed{19.5°}$

$$1.50 \sin\theta_3 = 1.00 \sin\theta_4$$
$$\theta_4 = \boxed{30.0°}$$



ANS FIG. P35.12

P35.16 (a) At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

or $1.00 \sin 30.0° = 1.50 \sin\theta_2$

$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00 \text{ cm}}{h}$$

or $h = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$

The angle of deviation upon entry is

$\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

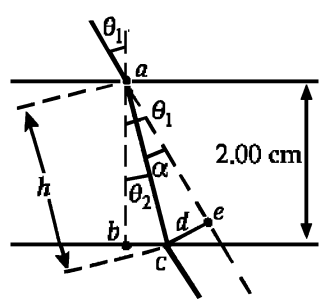
The offset distance comes from $\sin\alpha = \frac{d}{h}$: $d = (2.12 \text{ cm})\sin 10.5° = \boxed{0.387 \text{ cm}}$

(b) The speed of light in the material is

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$$

The distance h traveled by the light is $h = 2.12 \text{ cm}$. The time interval is

$$\Delta t = \frac{h}{v}$$
$$= \frac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = 1.06 \times 10^{-10} \text{ s} = \boxed{106 \text{ ps}}$$



ANS FIG. P35.16

P35.23 $n(700\text{ nm}) = 1.458$

(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2$; $\theta_2 = \boxed{41.5°}$

(b) Let $\theta_3 + \beta = 90.0°$, $\theta_2 + \alpha = 90.0°$

then $\alpha + \beta + 60.0° = 180°$

So

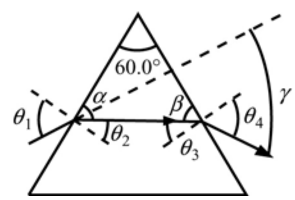
$$\alpha + \beta + 60.0° = 180°$$
$$(90.0° - \theta_2) + (90.0° - \theta_3) + 60.0° = 180°$$

$$60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$$

(c) $1.458\sin 18.5° = 1.00\sin\theta_4$ □ $\theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + (\theta_4 - \theta_3)$

$\gamma = (75.0° - 41.5°) + (27.6° - 18.5°) = \boxed{42.6°}$



ANS FIG. P35.23

P35.33 For the incoming ray, $\sin\theta_2 = \frac{\sin\theta_1}{n}$

Using the figure to the right,

$$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.66}\right) = 27.48°$$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray,

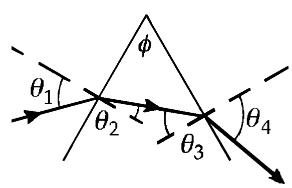
$$(90.0° - \theta_2) + (90.0° - \theta_3) + 60.0° = 180.0°$$
$$\theta_3 = 60.0° - \theta_2$$

and

$$\sin\theta_4 = n\sin\theta_3 : \quad (\theta_4)_{\text{violet}} = \sin^{-1}[1.66\sin 32.52°] = 63.17°$$

$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\sin 31.78°] = 58.56°$$

The angular dispersion is the difference
$\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$



ANS FIG. P35.33

P35.36 From Equation 35.10, $\sin\theta_c = \frac{n_2}{n_1}$, where n_2 = 1.000293. Values for n_1 come from Table 35.1,

(a) $\theta_c = \sin^{-1}\left(\frac{1.000293}{2.20}\right) = \boxed{27.0°}$

(b) $\theta_c = \sin^{-1}\left(\frac{1.000293}{1.66}\right) = \boxed{37.1°}$

(c) $\theta_c = \sin^{-1}\left(\frac{1.000293}{1.309}\right) = \boxed{49.8°}$

P35.43 (a) If any ray escapes it will be a ray along the inner edge, because it has the smallest angle of incidence. Its angle of incidence is described by $\sin\theta = \frac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

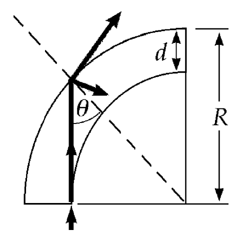
$$\frac{n(R-d)}{R} > 1 \qquad nR - nd > R \qquad nR - R > nd \quad R > \boxed{\frac{nd}{n-1}}$$

(b) As $d \to 0$, $\boxed{R_{min} \to 0}$. Yes: for very small d, the light strikes the interface at very large angles of incidence.

(c) As n increases, R_{min} decreases. Yes: as n increases, the critical angle becomes smaller.

(d) As n decreases toward 1, R_{min} increases. $R_{min} \to \infty$. Yes: as $n \to 1$, the critical angle becomes close to 90° and any bend will allow the light to escape.

(e) $R_{min} = \frac{1.40(100 \times 10^{-6}\ \text{m})}{0.40} = 350 \times 10^{-6}\ \text{m} = \boxed{350\ \mu\text{m}}$



ANS FIG. P35.43

P35.44 (a) In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark.

(b) To ensure total internal reflection at the plastic-air interface, the critical angle must be less than the angle of incidence, about 45.0°. This places a lower limit on the index of refraction of the plastic:

$$\theta_c \le 45.0°$$
$$\sin\theta_c \le \sin 45.0°$$
$$\frac{1}{n} \le \sin 45.0° \rightarrow \boxed{n \ge 1.41}$$

To prevent total internal reflection at the plastic-gasoline interface, the critical angle must be greater than the angle of incidence. This places an upper limit on the index of refraction of the plastic:

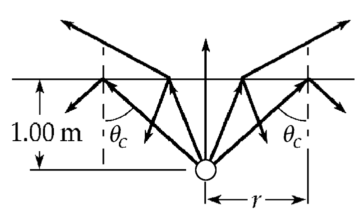
$$\theta_c \ge 45.0°$$
$$\sin\theta_c \ge \sin 45.0°$$
$$\frac{1.50}{n} \ge \sin 45.0° \rightarrow \boxed{n \le 2.12}$$

P35.45 For water, $\sin\theta_c = \frac{1}{4/3} = \frac{3}{4}$

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}$$



ANS FIG. P35.45

P35.47 From the textbook Figure P35.47, we have $w = 2b + a$

so
$$b = \frac{w-a}{2} = \frac{700\ \mu\text{m} - 1\ \mu\text{m}}{2} = 349.5\ \mu\text{m}$$
$$\tan\theta_2 = \frac{b}{t} = \frac{349.5\ \mu\text{m}}{1\,200\ \mu\text{m}} = 0.291 \qquad \theta_2 = 16.2°$$

For refraction at entry,
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\theta_1 = \sin^{-1}\frac{n_2\sin\theta_2}{n_1} = \sin^{-1}\frac{1.55\sin 16.2°}{1.00} = \sin^{-1} 0.433 = \boxed{25.7°}$$

***P35.54** The number N of reflections the beam makes before exiting at the other end is equal to the length of the slab divided by the component of the displacement of the beam for each reflection:

$$N = \frac{L}{(t/\tan\theta_2)} = \frac{L\tan\theta_2}{t}$$

where θ_2 is the refracted angle as the beam enters the material. Substitute for this refracted angle in terms of the incident angle by using Snell's law:

$$N = \frac{L}{t}\tan\left[\sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)\right]$$

Substitute numerical values:

$$N = \frac{0.420\ \text{m}}{0.003\,10\ \text{m}}\tan\left[\sin^{-1}\left(\frac{(1)\sin 50.0°}{1.48}\right)\right] = 81.96 \ \rightarrow\ 81\ \text{reflections}$$

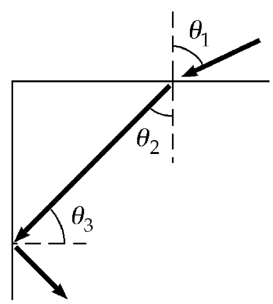
Therefore, the beam will exit after making 81 reflections, so it does not make 85 reflections.

P35.60 From Table 35.1, the index of refraction of polystyrene is 1.49.

(a) For polystyrene *surrounded by air*, total internal reflection requires

$$\theta_3 \geq \theta_c = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 \leq 47.8°$



ANS FIG. P35.60

From Snell's law,

$$\sin\theta_1 = 1.49\sin\theta_2 \leq 1.49\sin 47.8°$$
$$\sin\theta_1 \leq 1.10$$

Any angle θ_1 satisfies this equation.

Total internal reflection occurs for all values of θ, or the maximum angle is 90°.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

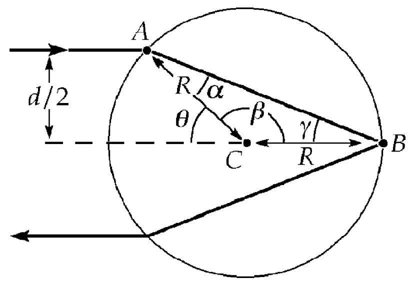
and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

From Table 35.1, the index of carbon disulfide is 1.628 > 1.49. Total internal reflection never occurs as the light moves from lower-index polystyrene into higher-index carbon disulfide.

P35.66 As shown in the sketch, the angle of incidence at point *A* is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$$



ANS FIG. P35.66

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline *CB* of the cylinder. In the isosceles triangle *ABC*,

$$\gamma = \alpha \text{ and } \beta = 180° - \theta$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

or $\alpha = \frac{\theta}{2} = 15.0°$

Then, applying Snell's law at point *A*,

$$n \sin\alpha = 1.00 \sin\theta$$
$$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

P35.74 (a) In the textbook figure, we have $r_1 = \sqrt{a^2 + x^2}$ and $r_2 = \sqrt{b^2 + (d-x)^2}$. The speeds in the two media are $v_1 = c/n_1$ and $v_2 = c/n_2$ so the travel time for the light from P to Q is indeed

$$\Delta t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2 + x^2}}{c} + \frac{n_2\sqrt{b^2 + (d-x)^2}}{c}$$

(b) Now $\frac{d(\Delta t)}{dx} = \frac{n_1}{2c}\frac{2x}{\sqrt{a^2 + x^2}} + \frac{n_2}{2c}\frac{2(d-x)(-1)}{\sqrt{b^2 + (d-x)^2}} = 0$ is the requirement for minimal travel time, which simplifies to $\frac{n_1 x}{\sqrt{a^2 + x^2}} = \frac{n_2(d-x)}{\sqrt{b^2 + (d-x)^2}}$.

(c) Now $\sin\theta_1 = \frac{x}{\sqrt{a^2 + x^2}}$ and $\sin\theta_2 = \frac{d-x}{\sqrt{b^2 + (d-x)^2}}$, so we have $n_1\sin\theta_1 = n_2\sin\theta_2$.