Physics 4C Mid-Term Exam 1, 2019 Fall

Instructions : There are 6 multiple choice questions and 2 longer problems. Read the problems *carefully* and give the best answer based on the material presented during class and in the text. The multiple choice questions are worth 60% (at 10% each) and the longer problems are worth 40% (at 20% each).

(1) A layer of water ($n = 1.333$) floats on a layer of carbon tetrachloride ($n = 1.461$). If light is traveling from the water into the carbon tetrachloride what is the critical angle at the interface (in degrees)? (a) 88 (b) 78 (c) 66 (d) 58 (e) the critcal angle is not defined

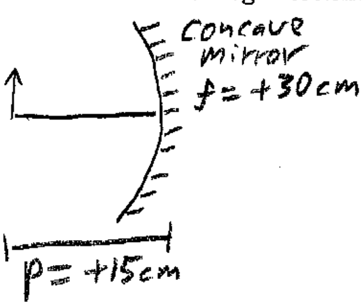
$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_2 \underbrace{\cancel{\sin 90^\circ}}_{=1} = n_2$

For total internal reflection,
$\theta_2 \geq 90^\circ$, but here,

$\theta_1 = \arcsin(n_2/n_1)$
$= \arcsin(1.461/1.333)$

$\boxed{\theta_1 \text{ is undefined} \Rightarrow \text{choice (e)}}$

(2) An object is placed 15 cm in front of a concave mirror with a focal length of 30 cm. What is the magnification? (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) -2



$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$

$= \frac{1}{30\text{cm}} - \frac{1}{15\text{cm}}$

$= \frac{1}{30\text{cm}} - \frac{2}{30\text{cm}}$

$\frac{1}{q} = -\frac{1}{30\text{cm}}$

$q = -30\text{ cm}$

$M = -\frac{q}{p} = \frac{-(-30\text{ cm})}{15\text{ cm}}$

$\boxed{M = +2 \Rightarrow \text{choice (b)}}$

(3) A lens has a convex front surface with a radius of curvature of 20 cm, and a back surface that is flat. It is made of glass, with n = 1.5. What is the focal length (in cm) of this lens?
(a) 20 (b) 30 (c) 40 (d) 10 (e) 50

$R_1 = +20\text{ cm}$

$R_2 = \infty$ (flat)

$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \overbrace{\cancel{\frac{1}{R_2}}}^{=0}\right]$

$= (1.5-1)\left[\frac{1}{20\text{cm}}\right] = \frac{0.5}{20\text{cm}}$

$f = \frac{20\text{ cm}}{0.5}$

$\boxed{f = +40\text{ cm} \Rightarrow \text{choice (c)}}$

(4) At a distance of 8 km from a radio transmitter, the amplitude of electric field strength is measured to be 0.35 V/m. What is the total power emitted by the transmitter? [Hint: The area of a sphere is $4\pi r^2$.]

(a) 1.63×10^{-4} W (b) 1.31×10^{5} W (c) 4.66×10^{-4} W (d) 3.74×10^{5} W (e) 16.38 W

Intensity $I = \frac{\text{Power}}{\text{Area}}$

$\Rightarrow \text{Power} = 4\pi R^2 I$

$R = 8\text{ km} = 8\times 10^3\text{ m}$

$I = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c}$ since $\frac{E_{max}}{B_{max}} = c$

$\text{Power} = \left(\frac{E_{max}^2}{2\mu_0 c}\right)(4\pi)(8\times10^3\text{ m})$

$\text{Power} = \frac{[(0.35)^2(4\pi)(8\times10^3)^2]}{[2(4\pi\times10^{-7})(3\times10^8)]}\text{ W}$

$\boxed{\text{Power} = 1.31\times10^5\text{ W} \Rightarrow \text{choice (b)}}$

(5) If the light from the Sun comes to Earth as a plane EM wave of intensity 1,340 W/m^2, calculate the peak values of E and B.

(a) 300 V/m, 10^{-4} T
(b) 1000 V/m, 3.35×10^{-6} T
(c) 225 V/m, 1.60×10^{-3} T
(d) 111 V/m, 3.00×10^{-5} T
(e) 711 V/m, 2.37×10^{-6} T

$I = 1340\text{ W/m}^2 = \frac{E_{max}B_{max}}{2\mu_0}$

$= \frac{E_{max}^2}{2\mu_0 c}$ since $\frac{E_{max}}{B_{max}} = c$.

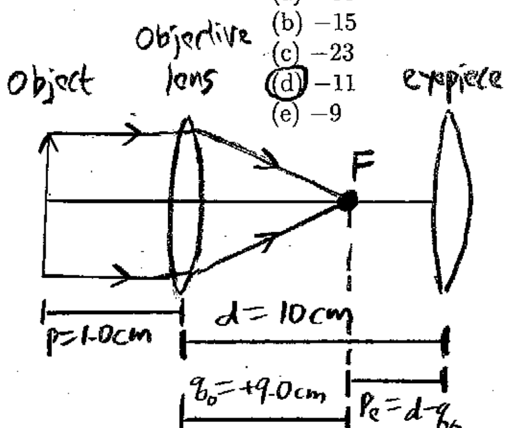
$E_{max} = \sqrt{2\mu_0 c I} = \left[2(4\pi\times10^{-7})(3\times10^8)(1340)\right]^{1/2}\frac{\text{V}}{\text{m}}$

$\boxed{E_{max} = 1.0\times10^3\text{ V/m}, \quad B_{max} = 3.35\times10^{-6}\text{ T} \Rightarrow \text{Choice (b)}}$

since $B_{max} = \frac{E_{max}}{c} = \frac{1000\text{ V/m}}{3\times10^8\text{ m/s}}$

(6) A microscope is made of two lenses. The one in front is called the objective lens, and the one in back is called the eyepiece. The objective lens has $f_0 = +0.90$ cm, and the eyepiece has $f_e = +1.1$ cm. The two lenses are separated by a distance of 10 cm. If an object is 1.0 cm in front of the objective lens, where (in cm) will the final image from the eyepiece be located?

(a) −30
(b) −15
(c) −23
(d) −11
(e) −9



Find q_e, the image distance for the 2nd lens, the eyepiece.

For any multi-lens system, the first lens (here q_0) has an image distance that equals the object distance of the second lens (here, p_e), so $q_0 = p_e$.

For the first lens (the objective lens),

$\frac{1}{p_0} + \frac{1}{q_0} = \frac{1}{f_0}$, so $\frac{1}{1.0\text{ cm}} + \frac{1}{q_0} = \frac{1}{+0.90\text{ cm}} \Rightarrow q_0 = +9.0\text{ cm}$

$p_e = d - q_0 = 10\text{ cm} - 9\text{ cm} = +1.0\text{ cm}$.

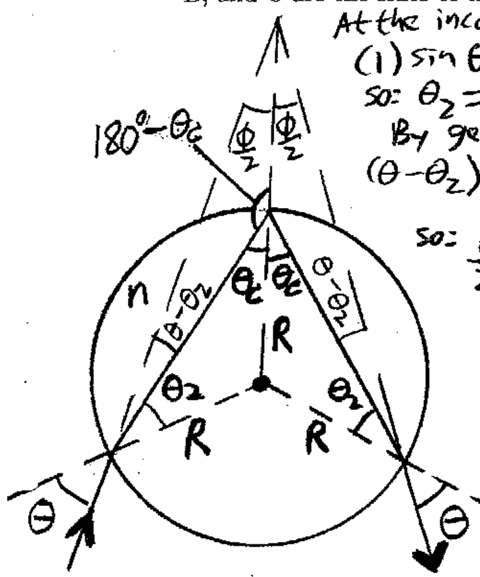
For the second lens (the eyepiece),

$\frac{1}{p_e} + \frac{1}{q_e} = \frac{1}{f_e}$, so $\frac{1}{+1.0\text{ cm}} + \frac{1}{q_e} = \frac{1}{+1.1\text{ cm}}$

$\Rightarrow \boxed{q_e = -11\text{ cm} \quad \text{Choice (d)}}$

Box your final answer. No work = No credit on this part.
Problems.

(A) (i) The figure on this page shows a ray of light passing through a spherical raindrop, with radius R. The raindrop has an index of refraction n, and the raindrop is surrounded by air, with n(air) = 1.00. The ray has an incident angle into the raindrop θ. Inside the raindrop, the ray undergoes two refractions and a total internal reflection, resulting in an angle ϕ between the incoming and the outgoing rays. Find ϕ as a function of θ and n only. [Hint: You may assume that the angle of the ray leaving the raindrop is θ. You may also use the law of sines, which for any triangle is $(\sin\alpha)/A = (\sin\beta)/B = (\sin\gamma)/C$, where α, β, and γ are the three angles of any triangle, and A, B, and C are the sides of the triangle that are opposite angles α, β, and γ, respectively.]



At the incoming refraction at left,

(1) $\sin\theta = n\sin\theta_2$

so: $\theta_2 = \arcsin\left(\frac{\sin\theta}{n}\right)$

By geometry,

$(\theta-\theta_2) + \frac{\phi}{2} + (\cancel{180^0}-\theta_c) = \cancel{180^0}$

so: $\frac{\phi}{2} - \theta_c + \theta = \theta_2$

By the law of sines,

$\frac{\sin\theta_c}{R} = \frac{\sin\theta_2}{R}$

so $\theta_c = \theta_2$

and $\theta_c = \arcsin\left(\frac{\sin\theta}{n}\right)$

Therefore,

$\phi = 2(\theta_2 + \theta_c - \theta)$

$= 2(2\theta_c - \theta)$

$= 4\theta_c - 2\theta$

and so:

$$\boxed{\phi = 4\arcsin\left(\frac{\sin\theta}{n}\right) - 2\theta}$$

(ii) All convex mirrors have $f < 0$. All convex mirrors form images in back of the mirror, so they have image distances $s' < 0$, which flat mirrors do too. For all mirrors,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f},$$

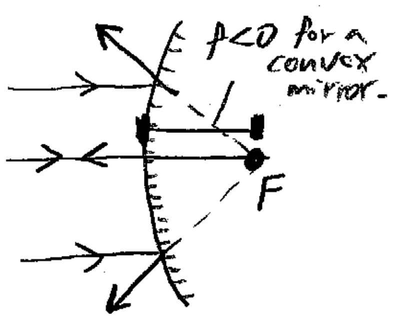
so: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ where $s = p$, $s' = q$, $f = f$.

where the object distance $s > 0$ always, since the object is in front of the mirror. Show that this implies that a convex mirror always shows an image that is upright and diminished.

By the sign conventions for mirrors, an upright image has $M > 0$.

$M = -\frac{q}{p} > 0$, since $q < 0$ and $p > 0$,

so the image in a convex mirror is <u>always upright.</u>



A diminished image has $|M| < 1$, which means $0 < M < 1$.

All convex mirrors have $f < 0$, so they have $1/f < 0$,

so: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} < 0$.

This implies:

$\frac{1}{p} < -\frac{1}{q}$

so: $\frac{q}{p} < -1$

and: $-\frac{q}{p} < +1$.

Since $M = -q/p$, $M < +1$.

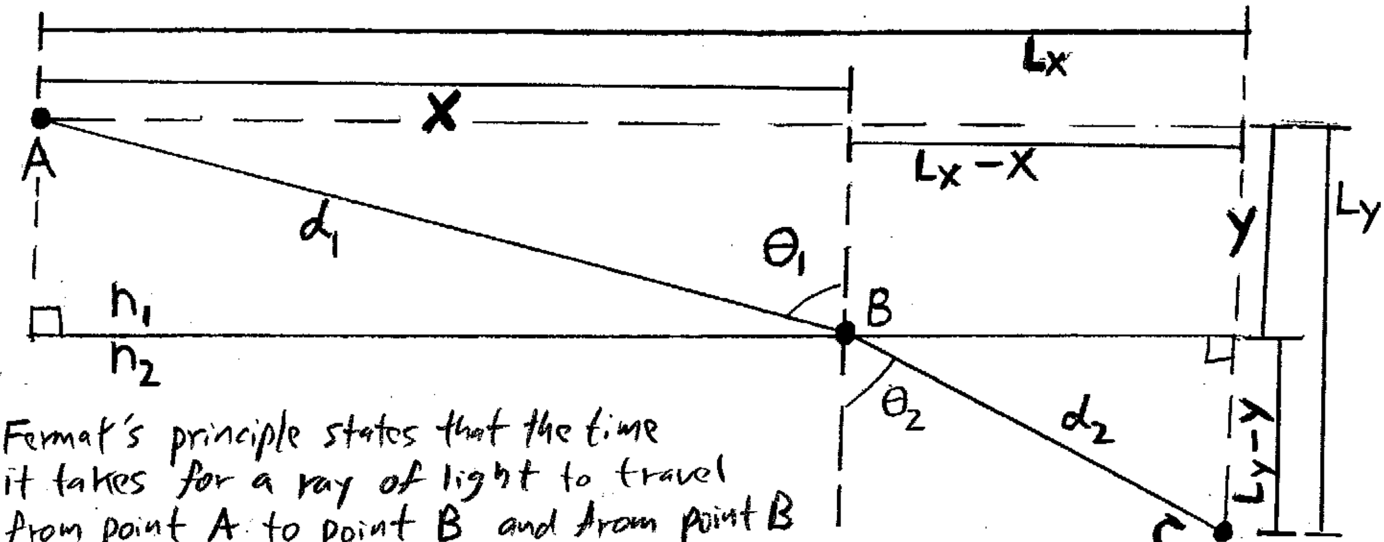
From at left, $M > 0$

so $0 < M < 1$

and so: $\boxed{|M| < 1}$,

so the image in a convex mirror is <u>always diminished.</u>

(B) A ray of light travels from point A to point B in a medium with index of refraction n_1, and then from point B to point C in another medium that has index of refraction n_2. Fermat's principle states that this ray of light travels from point A to point B, and then from point B to point C, in the minimum possible time. Use the figure on this page and a little geometry, trigonometry, and calculus to show that this implies Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. [Hint: recall that time = distance/speed, and that $n = c/v$. Recall also that to find the minimum time it takes a ray of light to travel a distance, take the first derivative with respect to distance, and set it equal to zero.]



Fermat's principle states that the time it takes for a ray of light to travel from point A to point B and from point B to point C must be a minimum, so:

$$t_{total} = t_{AB} + t_{BC} = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c}, \text{ since } n \equiv \frac{c}{v}.$$

$$t_{total} = \left[\frac{n_1\sqrt{x^2+y^2}}{c} + \frac{n_2\sqrt{(L_x-x)^2+(L_y-y)^2}}{c}\right]$$ since $d_1^2 = x^2 + y^2$ and: $d_2^2 = (L_x - x)^2 + (L_y - y)^2$

To find the minimum,

$$\frac{dt_{total}}{dx} = 0 = \left[\frac{2xn_1}{c\sqrt{x^2+y^2}} - \frac{2(L_x-x)n_2}{c\sqrt{(L_x-x)^2+(L_y-y)^2}}\right]$$

so: $$\frac{n_1 x}{\sqrt{x^2+y^2}} = \frac{n_2(L_x-x)}{\sqrt{(L_x-x)^2+(L_y-y)^2}}$$

$$\frac{n_1 x}{d_1} = \frac{n_2(L_x-x)}{d_2}$$

$$\therefore \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

This was a homework problem. Please do the homework carefully, and check the solutions carefully.