Phys 4C Mid-Term Exam 2 SOLUTIONS

(1) $\Delta x = 5.00 \times 10^{-11}\,m$

$\Delta x \Delta p_x \geq \hbar/2$, Heisenberg's Uncertainty Principle

$\Delta p_x = m(\Delta v)$

$(\Delta x)\, m(\Delta v) \geq \hbar/2$

$$\Delta v \geq \frac{\hbar}{2m(\Delta x)} = \frac{h}{4\pi m(\Delta x)}$$

$$\Delta v \geq \frac{(6.626\times 10^{-34}\,J\text{-}s)}{4\pi(9.11\times10^{-31}\,kg)(5.00\times10^{-11}\,m)}$$

$\boxed{\Delta v \geq 1.16\times 10^{6}\,m/s \Rightarrow \text{Choice (b)}}$

(2) De Broglie waves have: $\lambda = h/p$ and $p = h/\lambda$.
Non-relativistic kinetic energy $K = p^2/2m$.

$$\text{So: } K = \frac{h^2}{2m\lambda^2} = \frac{(6.626\times10^{-34}\,J\text{-}s)^2}{2(9.11\times10^{-31}\,kg)(1.0\times10^{-8}\,m)^2}$$

$\boxed{K = 2.4\times 10^{-21}\,J \Rightarrow \text{Choice (c)}}$

(3) For a relativistic electron, kinetic energ $K = (\gamma-1)m_e c^2$.
Here, $K = (\gamma-1)m_e c^2 = 2m_e c^2$, since $E = mc^2$ at rest, when $\gamma = 1$.

$(\gamma-1)\cancel{m_e c^2} = 2\cancel{m_e c^2}$

so $\gamma - 1 = 2$ and so $\gamma = 3$.

$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 3$, so $\frac{1}{1-(v/c)^2} = 9$, and: $\frac{1}{9} = 1-(v/c)^2$

so $(v/c)^2 = 8/9$ and $v = \sqrt{\frac{8}{9}}\,c$, so $\boxed{v = 0.94c \Rightarrow \text{Choice (c)}}$

(4) For Compton scattering, $\lambda' - \lambda_0 = \frac{h}{mc}(1-\cos\theta)$

For $\theta = 90^\circ$, $\lambda' - \lambda_0 = \frac{h}{m_e c}$

$$= \frac{(6.626\times10^{-34}\,J\text{-}s)}{(9.11\times10^{-31}\,kg)(3.0\times10^{8}\,m/s)}$$

$\boxed{\lambda' - \lambda_0 = 2.4\times10^{-12}\,m \Rightarrow \text{Choice (b)}}$

(5) For a quantum-mechanical particle in a box,

$$E = \left(\frac{h^2}{8mL^2}\right)$$

For $L' = L/2$, $\frac{E'}{E} = \left(\frac{L}{L'}\right)^2 = \left(\frac{L}{L/2}\right)^2 = 2^2$

so $\boxed{\frac{E'}{E} = 4}$ $\boxed{\text{choice (d)}}$, the particle's energy <u>quadruples</u>.

(6) $$P = 1 = \int_{-\infty}^{+\infty} |\psi|^2 dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx, \quad a = \frac{n\pi}{L}$$

$$1/A^2 = \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = \left[\frac{x}{2} - \frac{\sin(2n\pi x/L)}{(4n\pi/L)}\right|_0^L = \frac{L}{2}$$

$\therefore$ $\boxed{A = \sqrt{2/L} \Rightarrow \text{choice (a)}}$

(A) (i) $\theta_{min} = \frac{1.22\lambda}{D} \simeq \frac{Y}{L}$ for a circular aperture.

$$\Rightarrow y = \frac{1.22\lambda L}{D} = \frac{(1.22)(550\times10^{-9}m)(2\times10^{5}m)}{2.5m}$$

$$\boxed{Y = 0.054\,m = 5.4\,cm}$$

(ii) $K_{max} = hf - \phi = 0$

$$hf = \phi$$

$$\frac{hc}{\lambda} = \phi$$

$$\Rightarrow \lambda_{cutoff} = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}J.s)(3\times10^{8}m/s)}{1.125\,eV\left(\frac{1.602\times10^{-19}J}{1eV}\right)}$$

$$\boxed{\lambda_{cutoff} = 1.10\times10^{-6}m}\ \boxed{= 1100\,nm}$$

(iii) $\lambda_{max} = \frac{2.898\times10^{-3}\,m\cdot K}{T}$

$$T = \frac{2.898\times10^{-3}\,m\cdot K}{10\times10^{-6}m} = \frac{2.898\times10^{-3}\,m\cdot K}{(10^{-5}m)}$$

$$\boxed{T = 289.8\,K}$$

(B) (i) $L = \frac{L_p}{\gamma} = L_p\sqrt{1-(v/c)^2}$

$= (100\,m)\sqrt{1-(0.5c/c)^2} = (100\,m)\sqrt{1-(0.5)^2}$

$= (100\,m)\sqrt{1-(0.25)} = (100\,m)\sqrt{0.75}$

$\boxed{L = 86.6\,m}$

(ii) $u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{0.75c + 0.5c}{1 + \frac{(0.75c)(0.5c)}{c^2}} = \frac{1.25c}{1.375}$

$\boxed{u_x = 0.91c}$

(iii) $p = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}} = \frac{(1.0\times10^3\,kg)(0.5c)}{\sqrt{1-(0.5)^2}}$

$= \frac{(1.0\times10^3\,kg)(0.5)(3\times10^8\,m/s)}{0.866}$

$\therefore \boxed{p = 1.7\times10^{11}\ kg\ m\ s^{-1}}$

(iv) $E = \gamma m c^2 = \frac{(1.0\times10^3\,kg)(3\times10^8\,m/s)^2}{\sqrt{1-(0.5c/c)^2}}$

$\boxed{E = 1.0\times10^{20}\,J}$