

Phys 4C Mid-Term Exam 2 SOLUTIONS

(1) $\Delta x = 5.00 \times 10^{-11} \text{ m}$

$\Delta x \Delta p \geq \hbar/2$, Heisenberg's Uncertainty Principle

$\Delta p = m(\Delta v)$

$(\Delta x)m(\Delta v) \geq \hbar/2$

$$\Delta v \geq \frac{\hbar}{2m(\Delta x)} = \frac{h}{4\pi m(\Delta x)}$$

$$\Delta v \geq \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})}{4\pi (9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^{-11} \text{ m})}$$

$$\Delta v \geq 1.16 \times 10^6 \text{ m/s} \Rightarrow \text{choice (b)}$$

(2) De Broglie waves have: $\lambda = h/p \Rightarrow p = h/\lambda$.
Non-relativistic kinetic energy $K = p^2/2m$

$$\text{so: } K = \frac{h^2}{2m\lambda^2} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1 \times 10^{-8} \text{ m})^2}$$

$$K = 2.4 \times 10^{-21} \text{ J} \Rightarrow \text{choice (c)}$$

(3) When unpolarized light passes through a polarizer at any angle, it blocks half the light, so $I/I_{\text{max}} = 0.5 \Rightarrow \text{choice (c)}$

(4) For Compton scattering, $\lambda' - \lambda_0 = \frac{h}{mc} (1 - \cos \theta)$

For $\theta = 90^\circ$,

$$\lambda' - \lambda_0 = \frac{h}{mc}$$

$$= \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})}{(9.11 \times 10^{-31} \text{ kg})(3 \times 10^8 \text{ m/s})}$$

$$\lambda' - \lambda_0 = 2.4 \times 10^{-12} \text{ m} \Rightarrow \text{choice (b)}$$

(2)

(5) For a quantum-mechanical particle in a box,

$$E = \left(\frac{h^2}{8mL^2} \right)$$

$$\text{For } L' = L/2, \quad \frac{E'}{E} = \left(\frac{L}{L'} \right)^2 = \left(\frac{L}{L/2} \right)^2 = 2^2$$

so $\boxed{\frac{E'}{E} = 4}$

choice (d), the particle's energy quadruples.

$$(6) \quad P = 1 = \int_{-\infty}^{+\infty} |\psi|^2 dx = \int_0^L A^2 \sin^2 \left(\frac{n\pi x}{L} \right) dx, \quad a = \frac{n\pi}{L}$$

$$1/A^2 = \int_0^L \sin^2 \left(\frac{n\pi x}{L} \right) dx = \left[\frac{x}{2} - \frac{\sin(2n\pi x/L)}{(4n\pi/L)} \right]_0^L = \frac{L}{2}$$

$$\therefore \boxed{A = \sqrt{2/L} \Rightarrow \text{choice (a)}}$$

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(A) (i) $\theta_{\min} = \frac{1.22\lambda}{D} \approx \frac{y}{L}$ for a circular aperture.

$$\Rightarrow y = \frac{1.22\lambda L}{D} = \frac{(1.22)(550 \times 10^{-9} \text{ m})(2 \times 10^5 \text{ m})}{2.5 \text{ m}}$$

$$y = 0.054 \text{ m} = 5.4 \text{ cm}$$

(ii) $K_{\max} = hf - \phi = 0$

$$hf = \phi$$

$$\frac{hc}{\lambda} = \phi$$

$$\Rightarrow \lambda_{\text{cutoff}} = \frac{hc}{\phi} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{1.125 \text{ eV} \left(\frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right)}$$

$$\lambda_{\text{cutoff}} = 1.10 \times 10^{-6} \text{ m} = 1100 \text{ nm}$$

(iii) $\lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m}\cdot\text{K}}{T}$

$$T = \frac{2.898 \times 10^{-3} \text{ m}\cdot\text{K}}{10 \times 10^{-6} \text{ m}} = \frac{2.898 \times 10^{-3} \text{ m}\cdot\text{K}}{(10^{-5} \text{ m})}$$

$$T = 289.8 \text{ K}$$

(4)

$$\begin{aligned} \text{(B) (i)} \quad L &= \frac{L_p}{\gamma} = L_p \sqrt{1 - (v/c)^2} \\ &= (100\text{m}) \sqrt{1 - (0.5c/c)^2} = (100\text{m}) \sqrt{1 - (0.5)^2} \\ &= (100\text{m}) \sqrt{1 - (0.25)} = (100\text{m}) \sqrt{0.75} \\ &\boxed{L = 86.6\text{m}} \end{aligned}$$

$$\text{(ii)} \quad u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{0.75c + 0.5c}{1 + \frac{(0.75c)(0.5c)}{c^2}} = \frac{1.25c}{1.375}$$

$$\boxed{u_x = 0.91c}$$

$$\begin{aligned} \text{(iii)} \quad p &= \gamma m v = \frac{m v}{\sqrt{1 - (v/c)^2}} = \frac{(1.0 \times 10^3 \text{kg})(0.5c)}{\sqrt{1 - (0.5)^2}} \\ &= \frac{(1.0 \times 10^3 \text{kg})(0.5)(3 \times 10^8 \text{m/s})}{0.866} \end{aligned}$$

$$\therefore \boxed{p = 1.7 \times 10^{11} \text{kg m s}^{-1}}$$

$$\text{(iv)} \quad E = \gamma m c^2 = \frac{(1.0 \times 10^3 \text{kg})(3 \times 10^8 \text{m/s})^2}{\sqrt{1 - (0.5c/c)^2}}$$

$$\boxed{E = 1.0 \times 10^{20} \text{J}}$$