SOLUTIONS

Physics 4C Mid-Term Exam #2

Instructions: Read the problems *carefully* and give the best answer, based on the material presented during class and in the text. The multiple choice problems are worth 10% each, and the other problems are worth 20% each.

(1) Electrons (with mass 9.11×10^{-31} kg) are moving with a velocity of 30 m/s when they pass through a single slit of width 8.5×10^{-5} meters. Due to the wave nature of the electrons, a single-slit diffraction pattern is formed. At what angle (in degrees) are either of the first-order minima of this pattern located? (Hints: find the wavelength of the electrons, and notice that 30 m/s is non-relativistic.)
(a) 73.4 (b) 35.3 (c) 16.6 (d) 0.3 (e) No first-order minima are possible at this velocity. $a \sin\theta = m\lambda$ and $\lambda = \frac{h}{p} = \frac{h}{m_e v}$, non-relativistic since $v = 30\,\frac{km}{s} \ll c$.

So: $\sin\theta = \frac{mh}{m_e v a} = \frac{(1)(6.626\times10^{-34}\,J\cdot s)}{(9.11\times10^{-31}\,kg)(30\,\frac{m}{s})(8.5\times10^{-5}\,m)}$

$\sin\theta = 0.285$, so $\boxed{\theta = 16.6^\circ}$, $\boxed{\text{Choice (c)}}$

(2) A light source can determine the location of an electron in an atom to a precision of 7.5×10^{-9} meters. What is the minimum possible uncertainty in the speed (in m/s) of the electron? [Hint: remember that electrons in atoms are non-relativistic, with $v \ll c$.]
(a) 7.0 $\times10^{-27}$ (b) 5.7 $\times10^{-10}$ (c) 7.7 $\times10^{3}$ (d) 3.4 $\times10^{5}$ (e) 1.2×10^{6}

$\Delta x \Delta p_x \leq \hbar/2$

$\Delta x(m\,\Delta v) \leq \hbar/2$

$\Delta v \leq \frac{\hbar}{2(\Delta x)\,m}$

$\Delta v \leq \frac{h}{4\pi m(\Delta x)} = \frac{6.626\times10^{-34}\,J\cdot s}{4\pi(9.11\times10^{-31}\,kg)(7.5\times10^{-9}\,m)}$

$\boxed{\Delta v \leq 7.7\times10^{3}\,\frac{m}{s}}$ $\boxed{\to \text{Choice (c)}}$

(3) An electron is accelerated through a potential difference of 250 Volts. What is the de Broglie wavelength of the electron, in meters? Recall that electric potential energy is equal to voltage times charge, and assume non-relativistic motion.
(a) 1.1×10^{-17} (b) 7.8×10^{-11} (c) 8.5×10^{-24} (d) 4.0×10^{-19} (e) none of the above

$U = qV$, and since energy is conserved,

$\frac{p^2}{2m_e} = \Delta KE = \Delta U = qV$, so $p = \sqrt{2m_e qV}$.

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e qV}} = \frac{6.626\times10^{-34}\,J\cdot s}{[2(9.11\times10^{-31}\,kg)(1.602\times10^{-19}\,C)(250\,V)]^{1/2}}$

$\boxed{\lambda = 7.8\times10^{-11}\,\frac{m}{s}}$ $\boxed{\to \text{Choice (b)}}$

(4) If the temperature of a thermal radiator (also known as a black body) is doubled, then the wavelength at which the maximum intensity occurs will:
(a) double (b) quadruple (c) decrease by $\frac{1}{2}$ (d) decrease by $\frac{1}{4}$ (e) stay the same

Wien's law is: $\lambda_{max} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T(\text{K})}$, and here, $T_2 = T_1$.

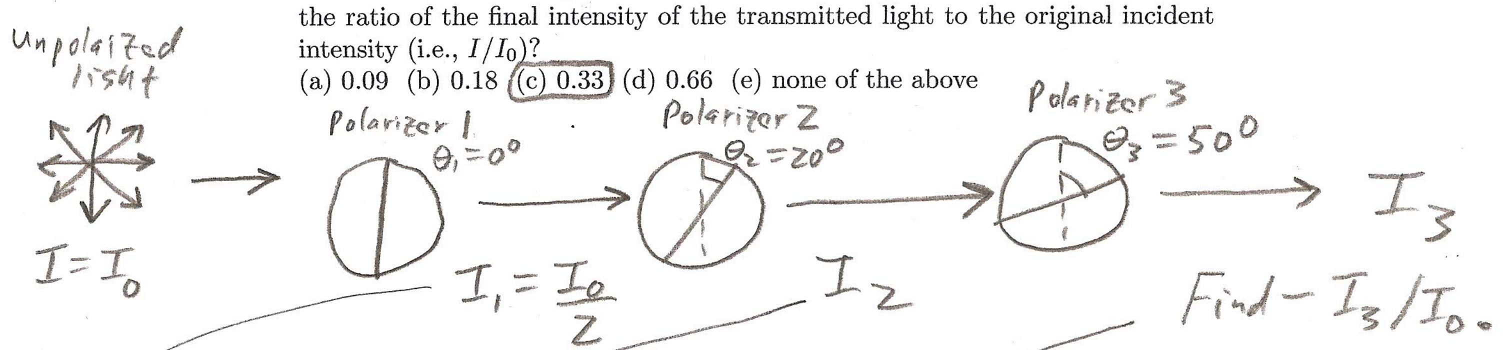
So: $\dfrac{\lambda_{max}(2)}{\lambda_{max}(1)} = \dfrac{T_1}{T_2} = \dfrac{T_1}{2T_1} = \boxed{\dfrac{1}{2} \rightarrow \text{choice (c)}}$

(5) Suppose a spaceship is traveling at $v = 0.75$ c, relative to a stationary observer. This spaceship launches a torpedo at a speed of $u_x' = 0.5$ c, relative to the spaceship, and in the same direction as the motion of the spaceship. What speed does the torpedo have, relative to the stationary observer?
(a) 3.333 c (b) 1.25 c (c) 0.993 c (d) 0.909 c (e) none of the above

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{0.5c + 0.75c}{1 + \frac{(0.5)(0.75)\cancel{c^2}}{\cancel{c^2}}} = \frac{1.25c}{1.375}$$

$$\boxed{u_x = 0.909c \rightarrow \text{choice (d)}}$$

(6) A beam of unpolarized light is incident on three polarizers. The first polarizer has its polarization axis vertical. The second polarizer has its axis 20 degrees to vertical. The third polarizer has its axis 50 degrees from vertical. What is the ratio of the final intensity of the transmitted light to the original incident intensity (i.e., I/I_0)?
(a) 0.09 (b) 0.18 (c) 0.33 (d) 0.66 (e) none of the above



$$I_2 = I_1 \cos^2(20^0)$$
$$I_3 = I_2 \cos^2(50^0 - 20^0) = I_2 \cos^2(30^0)$$
$$I_3 = I_1 \cos^2(20^0)\cos^2(30^0)$$
$$I_3 = \frac{I_0}{2}\cos^2(20^0)\cos^2(30^0)$$

$$\boxed{I_3 = 0.33 I_0 \rightarrow \text{choice (c)}}$$

Box your final answer. No work=No credit

Problems.
(A) A particle of mass m is confined inside a one-dimensional box of width L. One wall of the box is at $x = 0$ and the other is at $x = L$. The particle is the second excited state ($n = 3$) and is described by the wavefunction $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$.

(i) (10 points) What is the probability for finding this particle somewhere between $0 \le x \le \frac{L}{2}$? Use the integral $\int sin^2(ax)\, dx = \frac{x}{2} - \frac{sin(2ax)}{4a}$. Give the value and the explicit analytical solution, or in other words, the worked integration for this problem.

(ii) (5 points) What is the probability for finding this particle somewhere between $0 \le x \le L$? [Hint: there is an easy way to determine this.]

(iii) (5 points) If the particle makes a transition from $n = 3$ to the ground state, where $n = 1$, what is the energy of the photon that is emitted? Write your answer symbolically, in terms of m and L.

(i) $p = \int |\psi(x)|^2 dx = \int_{x=0}^{x=L/2} \frac{2}{L} \sin^2\left(\frac{3\pi x}{L}\right) dx$ $\quad a = \frac{3\pi}{L}$

$= \frac{2}{L} \int_0^{L/2} \sin^2(ax)\, dx = \frac{2}{L}\left[\frac{x}{2} - \frac{\sin(2ax)}{4a}\right]_{x=0}^{x=L/2}$

$= \frac{2}{L}\left[\frac{x}{2} - \frac{L \cdot \sin\left(\frac{6\pi x}{L}\right)}{12\pi}\right]_{x=0}^{x=L/2} = \frac{2}{L}\left[\frac{L}{4} - \frac{L}{12\pi}\underbrace{\sin(3\pi)}_{=0} - 0 - 0\right]$

$p = \frac{2}{L}\left(\frac{L}{4}\right) = \boxed{p = \frac{1}{2}}$ that the particle is between $x = 0$ and $x = L/2$.

(ii) The particle can't be outside the box, for which $x < 0$ and $x > L$, so $\boxed{p = 1}$ for $0 \le x \le L$.

(iii) $E = \left(\frac{h^2}{8mL^2}\right) n^2$

$E_{3\to 1} = \left(\frac{h^2}{8mL^2}\right)(3^2 - 1^2) \rightarrow E_{3\to 1} = \frac{\cancel{8} h^2}{\cancel{8} m L^2}$

$\therefore \boxed{E_{3\to 1} = \frac{h^2}{m L^2}.}$

(B) Lithium has a work function of 2.30 eV. Mercury has a work function of 4.50 eV.

(i) (10 points) What are the cutoff wavelengths for each metal, in meters? [Hint: you can save time by noting that $hc = 1240$ eV $\cdot$ nm.]

(ii) (5 points) If light with a wavelength of 400 nm is used, which of the metals will exhibit the photoelectric effect? Show your work, or no credit will be given.

(iii) (5 points) For the metal or metals here that exhibit the photoelectric effect, what is the maximum kinetic energy, in eV, of the emitted electrons when the 400 nm light is used?

(i) $K_{max} = \frac{hc}{\lambda} - \phi$

At cutoff,

$$0 = \frac{hc}{\lambda_c} - \phi$$

So for Li:

$$\lambda_c = \frac{hc}{\phi(Li)} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.30 \text{ eV}} = \boxed{\lambda_c(Li) = 539 \text{ nm}}.$$

For Hg:

$$\lambda_c = \frac{hc}{\phi(Hg)} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.50 \text{ eV}} = \boxed{\lambda_c(Hg) = 276 \text{ nm}}.$$

(ii) with light with $\lambda = 400$ nm,

for lithium, $K_{max} = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} - 2.30 \text{ eV}$

$K_{max} = 3.10 \text{ eV} - 2.30 \text{ eV} = 0.80 \text{ eV}$,

so lithium would show the photoelectric effect, for light with $\lambda = 400$ nm.

For mercury, $K_{max} = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} - 4.50 \text{ eV}$

$= 3.10 \text{ eV} - 4.50 \text{ eV} < 0$,

so mercury would not show the photoelectric effect, for light with $\lambda = 400$ nm.

(iii) For lithium,

$K_{max} = 0.80 \text{ eV}$, from (ii).