Physics 4C Practice Mid-Term Exam #2

Instructions: Read the problems *carefully* and give the best answer. The multiple-choice questions on this page are worth 10 points each.

(1) The location of an electron in an atom is known to a precision of 0.0500 nm $= 5.00 \times 10^{-11}$ m. The electron mass is 9.11×10^{-31} kg. What is the minimum possible uncertainty in the speed of the electron (in m/s)? [Hint: Non-relativistic formulas apply.]
(a) 1.46×10^7 (b) 1.16×10^6 (c) 7.27×10^6 (d) 2.32×10^6 (e) none of the above

(2) In an electron microscope, the de Broglie wavelength of the electrons must be equal to (or smaller than) the diameter of the object that is being viewed. Suppose one is trying to observe a large, spherical molecule that is 1.0×10^{-8} m in diameter. What is the minimum **kinetic** energy (in Joules) the electrons must have? [Hint: Non-relativistic formulas apply.]
(a) 6.6×10^{-26} (b) 4.4×10^{-51} (c) 2.4×10^{-21} (d) 1.0×10^{-8} (e) 3.6×10^4

(3) A relativistic electron (with $v > 0.1\ c$) has a kinetic energy equal to twice its rest energy. Determine its speed. (a) 0.76 c (b) 0.81 c (c) 0.94 c (d) 0.54 c (e) 0.87 c

(4) The Compton wavelength of the electron is equal to the shift in wavelength due to Compton scattering (in other words, $\Delta\lambda = \lambda' - \lambda_0$) when the scattering angle for the photon $\theta = 90$ degrees. The electron mass is 9.11×10^{-31} kg. What is the value of the Compton wavelength of the electron, in meters?

(a) 3.85×10^{-13} (b) 2.43×10^{-12} (c) 5.11×10^{-11} (d) 2.02×10^{-9} (e) 4.42×10^{-9}

(5) A quantum mechanical particle of mass m can only move in x, between $x = 0$ and $x = L$. If the size of the confining region is decreased by a factor of $\frac{1}{2}$, the energy of the particle will:

(a) decrease by $\frac{1}{2}$ (b) decrease by $\frac{1}{4}$ (c) double (d) quadruple

(6) The wavefunction of a particle confined to a one-dimensional box of length L is $\psi(x) = A \sin\left(\frac{n\pi x}{L}\right)$. The walls of the box are at $x = 0$ and $x = L$. The particle can't be outside the box, so $\psi(x) = 0$ for $x \leq 0$ and $\psi(x) = 0$ for $x \geq L$. Find the constant A. [*Hint:* A useful integral is $\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$.]

(a) $A = \sqrt{\frac{2}{L}}$ (b) $A = \sqrt{\frac{1}{L}}$ (c) $A = 0$ (d) $A = \frac{2}{L}$ (e) none of the above

Problems: Box your final answers. No work = No credit on this part.

(A) A satellite is in a low-Earth orbit at an altitude $L = 200$ km $= 2.0 \times 10^5$ m above the surface of Earth. Its camera has a circular aperture (which is a diameter, not a radius) of $D = 2.5$ meters.

(i) (10 points) The satellite's camera can detect visible light, which has a wavelength of 550 nm (where 1 nm $= 10^{-9}$ m). If the camera has adaptive optics, it can compensate for the blurring effects of Earth's atmosphere. However, diffraction imposes an absolute limit on the camera's image resolution, or in other words, the smallest detail the camera can see. How many meters in diameter (y) is the smallest object on Earth that this camera can resolve, or see clearly? [Hint: since the satellite is many kilometers from Earth, $\sin\theta \approx y/L$.]

(ii) (5 points) This camera is a digital camera, much like those that are commercially popular today. Digital cameras don't use photographic film to detect light: they use electronic CCDs, or charge-coupled devices. CCDs detect light by the photoelectric effect: photons that land on a silicon crystal knock electrons out of the crystal, and these electrons are caught and turned into images by digital electronics.

Silicon has a work function of 1.125 eV, where 1 eV = 1 electron volt $= 1.602 \times 10^{-19}$ J. What is the longest wavelength of light (in meters) that a CCD can detect? [Hint: the cutoff frequency occurs where the kinetic energy of the electrons drops to zero.]

(iii) (5 points) This satellite also has an infrared detector. It can detect thermal (also called blackbody) radiation, from objects that are not transparent to light, such as rocks, cars, or buildings. Suppose one of these objects radiates the maximum intensity of its blackbody radiation at a wavelength $\lambda_{max} =$ 10 microns $= 1.0 \times 10^{-5}$ m. What is the temperature of this object, in Kelvins?

(B) (i) (5 points) A spaceship, while stationary on Earth, is 100 m long. How long (in meters) does this spaceship look to observers on Earth, when it is moving away from Earth at a speed of 0.5c? [Hint: Assume Earth is stationary. This is a good approximation here, since the Earth's motion is nowhere close to relativistic—or in other words, nowhere close to being fast enough for relativistic effects become noticeable.]

(ii) (5 points) As the spaceship travels away from Earth at a speed of 0.5c, it launches a probe away from Earth. The probe is traveling at a speed of 0.75c *relative* to the spaceship. What velocity do observers on Earth measure for the probe?

(iii) (5 points) If the spaceship has a mass of 1.0×10^3 kg, what is the momentum of the spaceship, as observed from Earth, when the spaceship is moving away from Earth at 0.5c?

(iv) (5 points) What is the **total** energy of the spaceship, as observed from Earth, when the spaceship is moving away from Earth at 0.5c?