

1. The domain of the function  $f(x) = \frac{3x}{\sqrt{x+2}}$  is

- (a)  $\mathbb{R}$
- (b)  $x \neq 2$
- (c)  $x \geq -2$
- (d)   $x > -2$

$x + 2$  must be greater than or equal to 0 because it is under a square root. But  $x + 2$  cannot be 0 because that would force division by 0. Therefore  $x + 2 > 0$ , or  $x > -2$ .

2.  $\lim_{t \rightarrow 2} \frac{t}{|t-2|} =$

- (a)   $\infty$
- (b)  $-\infty$
- (c) 2
- (d) does not exist.

As  $t$  approaches 2 from both the right and the left,  $\frac{t}{|t-2|}$  becomes more and more positive (the numerator and denominator are both positive, and the denominator gets closer and closer to 0). Therefore the limit is  $\infty$ .

3. The graph of the function  $f(x) = \frac{x+3}{x+3}$

- (a) is identical to the graph of  $f(x) = 1$
- (b) has a vertical asymptote at  $x = -3$
- (c) has a vertical tangent at  $x = -3$
- (d)  has a hole at  $x = -3$

$\frac{x+3}{x+3} = 1$  whenever  $x \neq -3$ . So the graphs are identical except at  $x = -3$ , where  $\frac{x+3}{x+3}$  is undefined.

4. If the velocity of a bicycle at time  $t = 30$  is  $\lim_{h \rightarrow 0} \frac{(30 + h)^2 - \sqrt{30 + h} - (30^2 - \sqrt{30})}{h}$  feet per second, then the distance traveled by the bicycle after  $t$  seconds could be
- (a)  $2t - \frac{1}{2}t^{-1/2}$   
 (b)   $t^2 - \sqrt{t}$   
 (c)  $t^2 - \sqrt{30}$   
 (d)  $t^2 + \sqrt{t}$

Velocity is the derivative of distance. Therefore, if the velocity of the bicycle at  $t = 30$  is equal to the above limit, then the distance traveled after  $t$  seconds is a function whose derivative is equal to that limit, or  $s(t) = t^2 - \sqrt{t}$ .

5. For the graph of  $f(x)$  shown at right,\*
- (a)  $f(x)$  is continuous and differentiable at  $x = 2$   
 (b)  $f(x)$  is differentiable but not continuous at  $x = 2$   
 (c)   $f(x)$  is continuous but not differentiable at  $x = 2$   
 (d)  $f(x)$  is neither differentiable nor continuous at  $x = 2$

\* Please see me for the picture of the graph.

The graph of  $f(x)$  shows a cusp (corner) at  $x = 2$ , so it is continuous but not differentiable there.

6. If  $f(-3) = 1$ ,  $f(2) = 1$ ,  $g(2) = -3$ , and  $g(1) = -1$ , then  $(g \circ f)(2) =$
- (a) 2  
 (b) 1  
 (c)  -1  
 (d) -3

$$(g \circ f)(2) = g(f(2)) = g(1) = -1.$$

7. The height in feet at time  $t$  (in seconds) of a ball thrown upward is  $s(t) = 50t - 16t^2$ . The average velocity of the ball during the first 2 seconds is
- (a) -14 ft./s  
 (b) -7 ft./s  
 (c) 36 ft./s  
 (d)  18 ft./s

Average velocity is distance traveled divided by time elapsed, or

$$\begin{aligned} \frac{s(2) - s(0)}{2 - 0} &= \frac{[50 \cdot 2 - 16 \cdot 2^2] - [50 \cdot 0 - 16 \cdot 0^2]}{2} \\ &= \frac{100 - 64}{2} = 18. \end{aligned}$$

8.  $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2} =$

- (a) 0
- (b)  $\frac{1}{3}$
- (c)  $-\frac{1}{3}$
- (d) does not exist.

$\frac{x-1}{x^2+x-2} = \frac{x-1}{(x-1)(x+2)}$ , which is equal to  $\frac{1}{x+2}$  whenever  $x \neq 1$ . Since the value of the limit does not depend on what happens *at*  $x = 1$ , only *near*  $x = 1$ , the value of the limit, using the limit laws, is  $\lim_{x \rightarrow 1} \frac{1}{x+2} = \frac{1}{3}$ .

9. For the graph of  $f(x)$  shown at right\*, which of the following could be a graph of  $f'(x)$ ?

\* Please see me for an explanation of this question.

**BONUS.** (5 points) If  $f(x) = \begin{cases} 5 & x > 3 \\ |2x - 1| & x \leq 3 \end{cases}$  then the domain of  $f'(x)$  is

- (a)  $x \neq \frac{1}{2}$
- (b)  $x \neq 3$
- (c)  $x \neq \frac{1}{2}, x \neq 3$
- (d)  $\mathbb{R}$
- (e) none of these.

The graph of  $f(x)$  consists of three straight lines as shown below. The derivative  $f'(x)$  is not defined at  $x = \frac{1}{2}$  and  $x = 3$ . Therefore the domain of  $f'(x)$  is all real numbers except  $\frac{1}{2}$  and 3.

