

Math 75 Quiz 2 - v. 1(blue) Solutions
Sections 4.1, 4.3, 4.4, 4.5

1. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

The critical numbers of the function $f(x) = 2x^{1/5} - 1$ are

- (a) $(\frac{2}{5})^{5/4}$ only
(b) 0 and $(\frac{2}{5})^{5/4}$ only
(c) 0 only
(d) There are no critical numbers of $f(x)$.

The domain of $f(x)$ is all real numbers. The domain of $f'(x) = \frac{2}{5}x^{-5/4} = \frac{2}{5x^{5/4}}$ is $x \neq 0$. Therefore $x = 0$ is a critical number of $f(x)$. Since there is no x -value for which $f'(x) = 0$, there are no other critical numbers of $f(x)$.

2. **Multiple Choice.** (5 points) *Circle the letter of the best answer.*

The function $g(x) = -x^2 + 6x + 1$

- (a) has a local minimum at $x = 3$
(b) has a local maximum at $x = 3$
(c) has an inflection point at $x = 3$
(d) none of these.

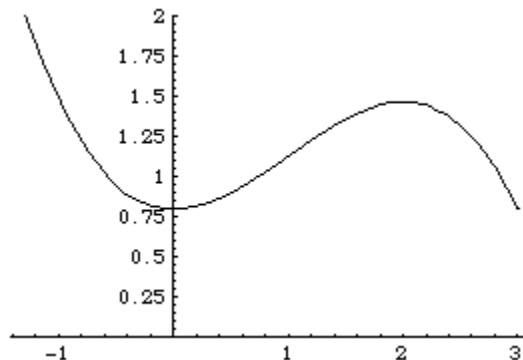
$g'(x) = -2x + 6$, which is 0 when $x = 3$. Therefore 3 is a critical number of $f(x)$.

Since $g(x)$ is a parabola opening down, the critical number must be the location of a local maximum.

Alternatively, we can test the intervals $(-\infty, 3)$ and $(3, \infty)$. Since $g'(0) = 6 > 0$, we know $g(x)$ is increasing on the interval $(-\infty, 3)$. Since $g'(4) = -2 < 0$, we know $g(x)$ is decreasing on the interval $(3, \infty)$.

3. **Graph.** (5 points) On the axes below, sketch the graph of a function $f(x)$ satisfying all of the following:

- $f(x)$ is decreasing for all $x < 0$.
- $f(x)$ is concave up for all $x < 1$
- $f(x)$ has an inflection point at $x = 1$
- $f(x)$ has a local maximum at $x = 2$



Answers will vary.

4. (10 points) Find $\lim_{x \rightarrow \infty} \frac{x^5 - 5x^4 + 1}{-2x^5 + 3x}$.

Show all steps!

We have

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^5 - 5x^4 + 1}{-2x^5 + 3x} &= \lim_{x \rightarrow \infty} \frac{x^5 - 5x^4 + 1}{-2x^5 + 3x} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}} \\ &= \lim_{x \rightarrow \infty} \frac{1 - \frac{5}{x} + \frac{1}{x^5}}{-2 + \frac{3}{x^4}} \\ &= \frac{1 - 0 + 0}{-2 + 0} \\ &= \boxed{-\frac{1}{2}} \end{aligned}$$

BONUS. (2 points) If the derivative of $f(x)$ is $f'(x) = \frac{x^2 - 5}{x - 3}$, find the x -coordinate(s) of the inflection point(s) of $f(x)$.

The inflection points of $f(x)$ occur at x -values for which $f''(x) = 0$. Since $f'(x) = \frac{x^2 - 5}{x - 3}$, we have

$$\begin{aligned} f''(x) &= \frac{(x - 3)2x - (x^2 - 5)}{(x - 3)^2} \\ &= \frac{2x^2 - 6x - x^2 + 5}{(x - 3)^2} \\ &= \frac{x^2 - 6x + 5}{(x - 3)^2} \stackrel{\text{set}}{=} 0 \\ x^2 - 6x + 5 &= 0 \\ (x - 1)(x - 5) &= 0 \\ \boxed{x = 1, \quad x = 5} \end{aligned}$$