Math 75 Practice for Quiz 2 - Solutions
Sections 4.1, 4.3, 4.4, 4.5

1. **Multiple Choice.** *Circle the letter of the best answer.* The critical numbers of the function $f(x) = \sqrt[3]{x^2 - 1}$ are

   (a) 0 only

   (b) 1 and -1 only

   (c) $\boxed{\text{0, 1, and } -1 \text{ only}}$

   (d) There are no critical numbers of $f(x)$.

   The domain of $f(x)$ is all real numbers. We have

$$\begin{aligned} f'(x) &= \tfrac{1}{3}(x^2 - 1)^{-2/3}(2x) \\ &= \frac{2x}{3(x^2 - 1)^{2/3}}, \end{aligned}$$

   which is undefined for $x = 1$ and $x = -1$ and is equal to 0 when $x = 0$.

2. **Multiple Choice.** *Circle the letter of the best answer.* The function $g(x) = \dfrac{\sin x}{x}$

   (a) is an odd function

   (b) $\boxed{\text{is an even function}}$

   (c) is neither an odd nor an even function

   We have

$$\begin{aligned} g(-x) &= \frac{\sin(-x)}{-x} \\ &= \frac{-\sin x}{-x} && \text{(since } \sin x \text{ is an odd function)} \\ &= \frac{\sin x}{x} = g(x). \end{aligned}$$

   Therefore $g(x)$ is an even function.

3. **Fill-In.** For the function $h(x) = \frac{5x^2}{1-2x^2}$,

the equation(s) of the vertical asymptote(s) is/are $\boxed{x = \sqrt{\frac{1}{2}},\ x = -\sqrt{\frac{1}{2}}}$

and the equation(s) of the horizontal asymptote(s) is/are $\boxed{y = -\frac{5}{2}}$.

Notice that $h(x)$ has vertical asymptotes whenever the denominator is 0, since there are no common factors to cancel. We have

$$\begin{aligned} 1 - 2x^2 &= 0 \\ 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \pm\sqrt{\frac{1}{2}}. \end{aligned}$$

For the horizontal asymptotes, we compute the limits at infinity of $h(x)$:

$$\lim_{x\to\pm\infty} \frac{5x^2}{1-2x^2} = -\frac{5}{2},$$

since $h(x)$ is a rational function whose top and bottom degrees match, and therefore the limit at infinity is equal to the leading coefficient of the top polynomial, divided by the leading coefficient of the bottom polynomial.

4. **Fill-In.** The inflection point of the function $j(x) = \frac{2}{3}x^3 - 2x^2$ is $\boxed{\left(1, -\frac{4}{3}\right)}$.

We have

$$j'(x) = 2x^2 - 4x,$$

so

$$j''(x) = 4x - 4.$$

Setting $j''(x) = 0$, we get $x = 1$. This is the x-coordinate of the inflection point. To get the y-coordinate, we plug in $x = 1$ to $j(x)$ and get

$$j(1) = \tfrac{2}{3} - 2 = -\tfrac{4}{3}.$$

5. Find the absolute maximum and minimum values of the function $f(x) = -5x^2 - 11x + 4$ on the interval $[-2, 0]$.

$f(x)$ is continuous on the interval $[-2, 0]$, so the Extreme Value Theorem applies. Therefore we only need to check the endpoints of the interval and the critical numbers inside the interval. We have

$$f(0) = 4,$$
$$f(-2) = -5(-2)^2 - 11(-2) + 4 = 6.$$

Also

$$\begin{aligned} f'(x) &= -10x - 11 \overset{\text{set}}{=} 0 \\ 10x &= -11 \\ x &= -\frac{11}{10}, \end{aligned}$$

which is a critical number in the interval $[-2, 0]$. Since $f(-\frac{11}{10}) = -5\left(-\frac{11}{10}\right)^2 - 11(-\frac{11}{10}) + 4 = \frac{1005}{100} = 10.05$, we see that the largest of the three values 4, 6, and 10.05 is 10.05 and the smallest is 4. Therefore the absolute maximum value is 10.05 and the absolute minimum value is 4.

6. Discuss the symmetry (or lack of it) of the function $g(x) = \dfrac{3x}{x^2 + 1}$.

To determine whether a function is even, odd, or neither, we compute $g(-x)$:

$$\begin{aligned} g(-x) &= \frac{3(-x)}{(-x)^2 + 1} \\ &= \frac{-3x}{x^2 + 1} \\ &= -\frac{3x}{x^2 + 1} \\ &= -g(x). \end{aligned}$$

Therefore $g(x)$ is an odd function, and its graph is symmetric about the origin.